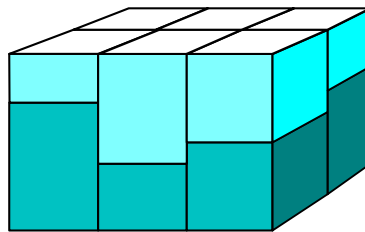


# **GRABLOX2**

Gravity interpretation and modelling using 3-D block models

User's guide to version 2.1



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## 1. Introduction

The measured gravity field is caused mainly by the gravitational attraction of Earth's mass, but it's also affected by the centrifugal force due to Earth's rotation and the ellipsoidal shape of the Earth. The international gravity formula (IGF) defines the normal value of the gravity on a reference ellipsoid the surface of which coincides with the mean sea level of the oceans. The gravity field is not uniform, because the mass is not uniformly distributed inside the Earth. Large scale anomalies in the gravity field and the potential defined by the geoid are caused by mass variations deep inside Earth's crust and mantle. Local anomalies that are usually studied in applied geophysics are caused by topography variations and near-surface mass distributions.

The (petrophysical) parameter considered in gravity studies is *density*,  $\rho$ , defined as the mass per volume unit [ $\text{kg/m}^3$  or  $\text{g/cm}^3$ ]. Density variations are related to variations in the material properties (structure and composition) of the rocks and soil. Therefore, gravity measurements provide an indirect method to study the geological structure of the Earth.

In applied geophysics the relative changes in the *vertical gravitational acceleration*,  $g_z$  [ $\text{m/s}^2$  or gal, for which  $1 \text{ milli-gal} = 10^{-5} \text{ m/s}^2$ ] between different sites are measured using (spring) gravimeters. More accurate absolute measurements made for geodetic purposes provide reference points in which the relative measurements can be tied to. The results are usually represented in a form of Bouguer anomaly, where the data have been reduced to the sea level using IGF and corrected for elevation changes (free-air correction) and the mass between the sea level and measurement point (Bouguer correction). Topographic and isostatic corrections are sometimes made also. Although, the amplitude of gravity anomalies can be hundreds of mgals in large scale studies, they can be less than one mgal in small-scale studies. Therefore, accurate instruments, knowledge on the elevation (and topography), and precise data correction methods are needed in gravity studies.

Gravity field is a vector field ( $\mathbf{F} = g_x\mathbf{i} + g_y\mathbf{j} + g_z\mathbf{k}$ ) which in source free region can be expressed as the gradient of the gravity potential,

$$\mathbf{F} = \nabla\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j} + \frac{\partial\phi}{\partial z}\mathbf{k}. \quad (1)$$

The gravity gradients that can be measured also are

$$g_{xx} = \frac{\partial^2\phi}{\partial x^2}, g_{xy} = \frac{\partial^2\phi}{\partial x\partial y}, g_{xz} = \frac{\partial^2\phi}{\partial x\partial z}, g_{yy} = \frac{\partial^2\phi}{\partial y^2}, g_{yz} = \frac{\partial^2\phi}{\partial y\partial z}, \text{ and } g_{zz} = \frac{\partial^2\phi}{\partial z^2}. \quad (2)$$

These so-called gravity tensor components are measured in Bell Geospace AirFTG system, whereas Falcon AGG system measures the gravity curvature component  $g_{uv} = (g_{xx} - g_{yy})/2$ .

The gravity potential of a mass distribution  $M$  within volume  $V$  can be defined as

$$\phi(\mathbf{r}) = G \frac{M}{|\mathbf{r} - \mathbf{r}'|} = G \int_V \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV', \quad (3)$$

where  $G \approx 6,673 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$  is the universal gravity constant,  $\mathbf{r} = (x-x_0)\mathbf{i} + (y-y_0)\mathbf{j} + (z-z_0)\mathbf{k}$  is position vector of the observation point,  $\mathbf{r}'$  is position vector of the source point inside the integration volume  $V$ , and  $\rho(\mathbf{r}')$  is a density distribution the volume integral of which is equal to the mass. The vertical component of the anomalous gravity field that is usually computed is

$$\Delta g(\mathbf{r}) = g_z(\mathbf{r}) = G \frac{\partial}{\partial z} \int_V \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV'. \quad (4)$$

If the density is constant inside the volume it can be taken out from the integral. In that case, the vertical component of the gravity gradient, for example, is

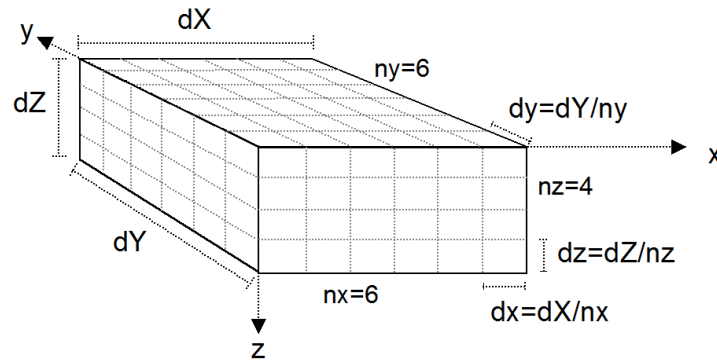
$$g_{zz}(\mathbf{r}) = G\rho \frac{\partial^2}{\partial z^2} \int_V \frac{1}{|\mathbf{r} - \mathbf{r}'|} dV'. \quad (5)$$

The unit of gravity gradient is  $[1/\text{s}^2]$ . However, just like mgal is used for gravity data because of the small amplitude of the geophysical anomalies, Eötvös [ $1 \text{ Eo} = 10^{-9} \text{ s}^{-2}$  or  $0,1 \text{ mgal/km}$ ] is used for gravity gradient data. For more information about gravity method in applied geophysics, please, see Blakely (1995) or any text book of applied geophysics.

The objective of gravity interpretation is to determine the density contrast and/or the shape and dimensions of the density variations. The general inverse problem does not have a unique solution. Instead, numerical modelling and inversion (parameter optimization) can be only used to construct a possible density distribution that could explain the measurements. Special

care must be taken when assessing the geological validity of the geophysical interpretation model since because of the non-uniqueness multiple different models can explain the data equally well. Recently airborne gravity gradient measurements have gained popularity. Since the gravity field and its gradients have different dependency on distance, the gradient data provide valuable constraining information that can improve the inversion result.

Figure 1.1 illustrates the three dimensional (3-D) block model (a.k.a. voxel model) used in GRABLOX. The rectangular main block representing the volume below the measurement area is divided into smaller brick-like volume elements. Each minor block is assigned constant density value,  $\rho_{ijk}$ . The gravity field is then computed as the superposition (sum) of all the minor blocks using equation (4). For more information about 3-D block models, please, see the documentation of the BLOXER program, which can be used to visualize and to edit the 3-D block models.



*Figure 1.1 Block model of size  $dX \times dY \times dZ$  (distance units) divided into  $N = nx \times ny \times nz$  minor blocks of size  $dx \times dy \times dz$  (distance units). The model is aligned with the (geographical) coordinate system.*

GRABLOX2 can be used for both forward and inverse modelling (inversion). Gravity field ( $g_z$ ) can be modelled with or without the vertical gravity gradient ( $g_{zz}$ ) or any combination of the six gravity tensor components ( $g_{xx}$ ,  $g_{xy}$ ,  $g_{xz}$ ,  $g_{yy}$ ,  $g_{yz}$ , and  $g_{zz}$ ) used in Bell FTG system, or the curvature component ( $g_{uv} = (g_{xx} - g_{yy})/2$ ) used in Falcon AGG system. The inversion method optimizes the density of the blocks so that the difference between the measured and the computed gravity (and gradient) data gets minimised. The optimization is based on linearised inversion. The unconstrained inversion uses singular value decomposition (SVD) with adaptive damping. The constrained inversion utilizes Occam's method where the roughness of the model is minimised together with data misfit. The coefficients of the base anomaly, which is represented by a second degree polynomial, can be optimised separately for the gravity and gradient data. Density of the blocks can be fixed (and weighted) based on a priori information

(e.g., petrophysical or drill-hole data). Gradient data can be used together with gravity data in the inversion, but gradient data cannot be inverted alone. After density inversion the distribution of the density variations inside the resulting block model can be used in geological interpretation.

### *1.1 About this version*

GRABLOX2 has limited abilities for model maintenance. BLOXER is a separate, free computer program that can be used for more enhanced model editing and visualization (<https://wiki.oulu.fi/x/eYU7AQ>). BLOXER can be used, for example, to import a priori density and topographic data into the model, export density values along vertical cross-sections of the model, and to interactively edit density, fix/free and hidden/visible status of the blocks. For even more advanced visualization of the 3D density models Golden Software's Voxler is highly recommended.

GRABLOX2 cannot be used to optimize the height of the blocks. To perform two-layer inversion for sediment and soil thickness, please, use the older GRABLOX version 1.7 (<https://wiki.oulu.fi/x/EoU7AQ>).

The 64 bit version of GRABLOX2 now contains OpenMP parallelisations that require the presence of Libiomp5md.dll dynamic link library.

## 2. Installing the program

GRABLOX2 can be run on a PC with Microsoft Windows operating system and a graphics display with a screen size of at least  $1280 \times 1024$  pixels. In forward modelling the computer memory and CPU requirements are usually not critical factors, because the program uses dynamic memory allocation and the computations are quite simple. However, models with hundreds of thousands of elements may take several hours to compute even on a modern computer. Likewise, inversion of block models with tens of thousands of elements may not be practical because of the increased computation time.

The size of continuous memory (1 GB) that is allocated for the sensitivity matrix restricts the usage of the 32 bit version. The 64 bit version of GRABLOX2 that can only be run under 64 bit Windows does not have this memory restriction. Furthermore, the 64 bit version uses OpenMP parallelisations and therefore runs faster on modern PC's with multiple processor cores.

GRABLOX2 has simple graphical user interface (GUI) that can be used to modify the parameter values, to handle file input and output, and to visualize the gravity data and the model. The user interface and graphics are based on the DISLIN graphics library (<http://www.dislin.de>).

The program requires either the 32 bit stand-alone version GRABLOX2.EXE or the 64 bit version GRABLOX64.EXE and the LIBIOMP5MD.DLL dynamic link library for OpenMP parallel computations. The distribution file (GRABLOX2.ZIP) also contains a short description file (\_README.TXT), user's manual (GRABLOX2\_MANU.PDF) in PDF format, and some example data (\*.DAT & \*.GAT) and model files (\*.INP & \*.BLX).

To install the program simply decompress the distribution ZIP file somewhere on a hard disk and a new folder GRABLOX2 appears. When running the program over local network, one should provide the network drive with a logical drive letter (e.g., Explorer/Tools/ Map network drive...).

### 3. Getting started

After startup the program shows a standard file selection dialog, which is used to provide the name of the input model file (\*.INP). If the user cancels the file selection operation the GRABLOX2.INP file will be used. If this file does not exist default parameters are used and the file will be created automatically. The parameters of individual blocks are read from a separate \*.BLX file. Before the user interface is built up, the program reads graph parameters from the GRABLOX2.DIS file. If this file does not exist default parameters are used and the file is created automatically. The program runs in two windows: 1) the console (shell) window and 2) the graphical user interface (GUI) window. The console window provides runtime information about the current status of the program. The information appearing on the console window is also stored in a log file (GRABLOX2.LOG).

After the initial model and the system parameters have been defined the user interface shown in Figs. 3.1 is build up and a density map of the top layer is shown (cf. Appendix A1). The model can be visualised using *Layers* and *Sections* push buttons. Repetitive use of *Layers* and *Sections* push buttons will show successive layers or sections. *Crossing dir* button swaps between (West-East directed) X-sections and (South-North directed) Y-sections. The <- / -> button is used to reverse the direction of layer and section swapping.

*Compute* button is used to perform forward computation. *Map data* button plots the data as a 2-D coloured contour or image map along with a 3-D view of the model and a short description on some parameters (see Appendix B1). If the computation of gravity gradients is activate, *Gravity/Gradient* button swaps the map between the two data types and *Grad. component* button swaps the map between the six tensor components. *Profiles* (and *Crossing dir*) button will display a single X- or Y-directed (interpolated) profile across the data area. Model and system parameters are changed using the text fields on the left control panel. *Update* button must be used to validate the changes. The computational settings can be changed using the items in the *Gravity* menu. Most of the controls on the right control panel are inactive before measured data have not been read in. To learn more about data interpretation, please, refer to chapters 5 and 6.



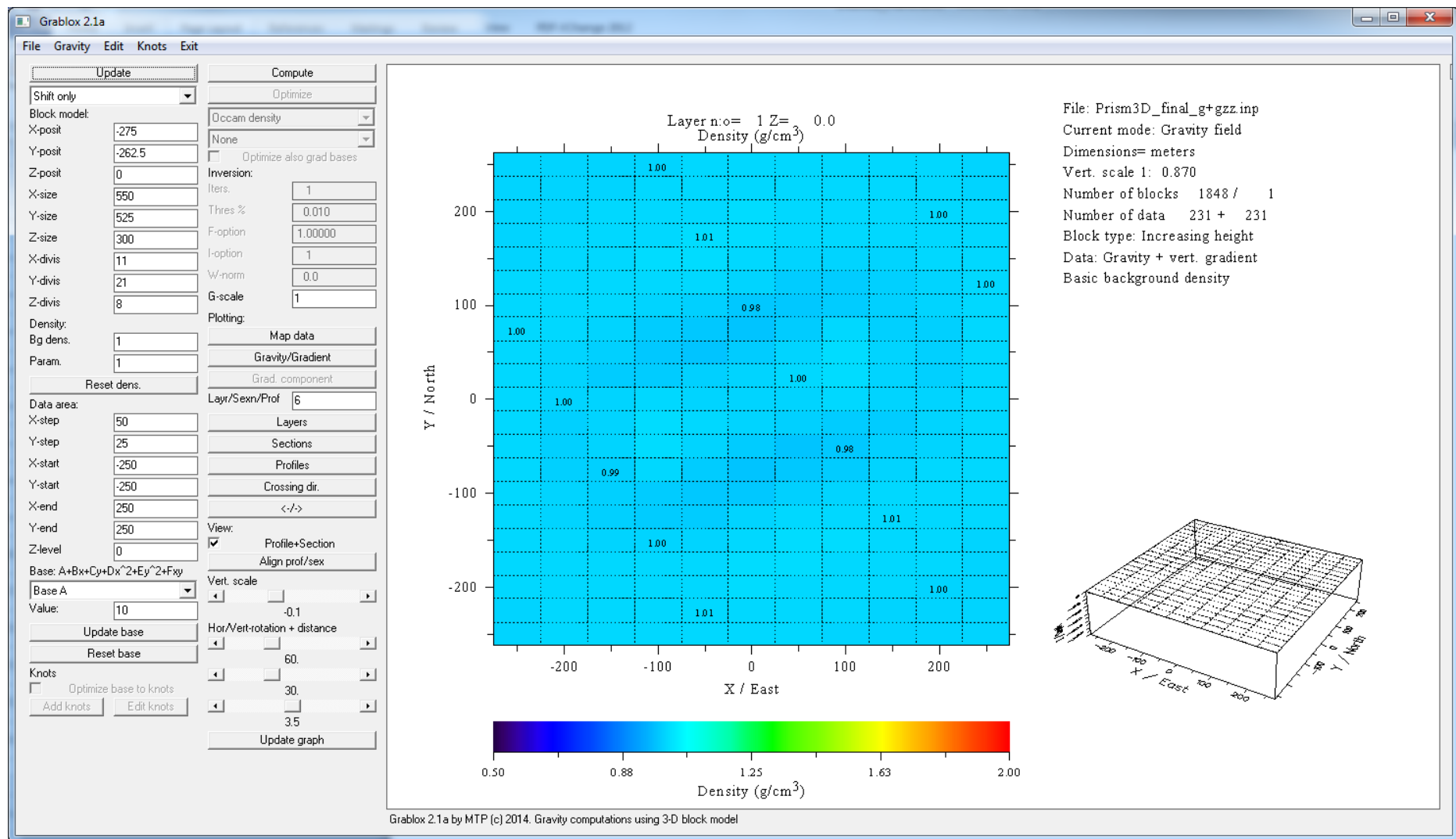


Figure 3.1. Grablox2 GUI with a layer

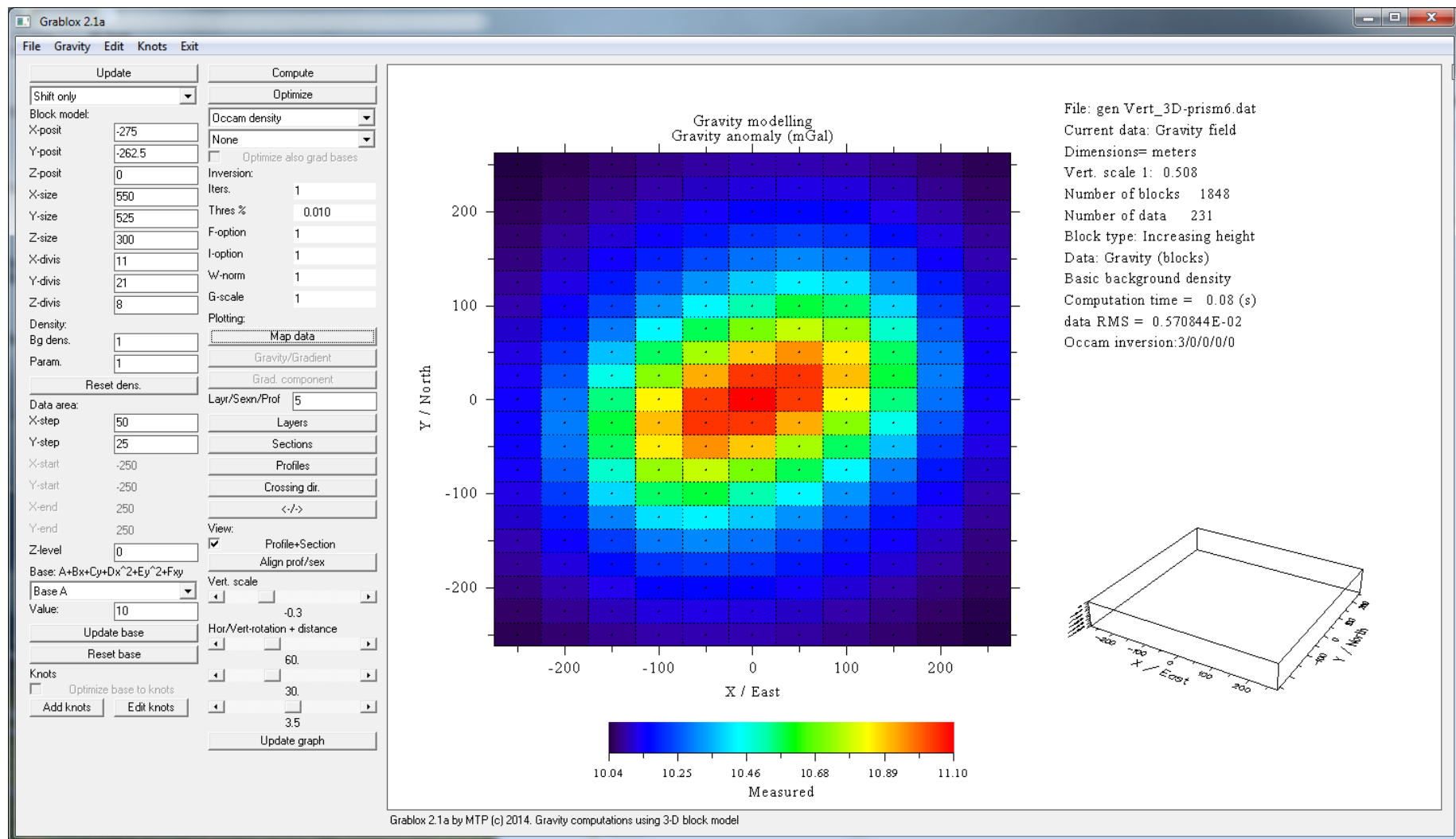


Figure 3.2. Image (pixel) map of the measured data.

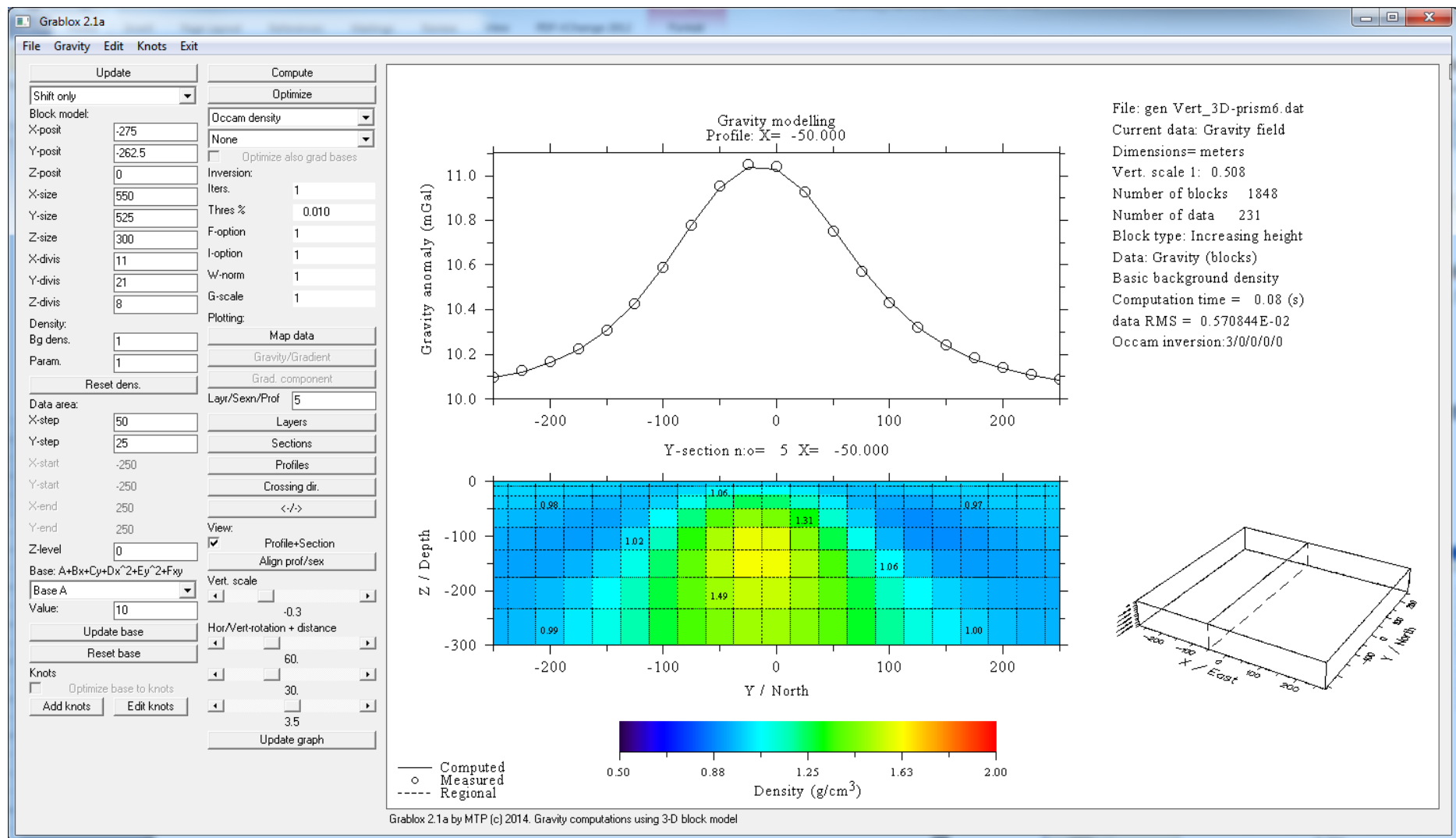


Figure 3.2. Section view of the density model and the fit between measured and modelled data

## 4. The user interface

### 4.1 File menu

Open model	Open an existing model file (*.INP)
Read grav. data	Read in gravity data for interpretation and comparison (*.DAT)
Read regional grav.	Read in regional data (stationary gravity field) (*.DAT)
Read gradient data	Read in gravity gradient data (*.GAT)
Read grad. regional	Read in regional trend of gravity gradient (*.GAT)
-----	
Save model	Save the model into a file (*.INP)
Save data	Save the data (computed & measured) into a file (*.DAT)
Save gradient data	Save the gradient data (computed & measured) into a file (*.GAT)
Save results	Save the results (description & data) into a file (*.OUT)
Read disp.	Read in new graph parameters from GRABLOX2.DIS
-----	
Save graph as PS	Save the graph in Adobe's Postscript format
Save graph as EPS	Save the graph in Adobe's Encapsulated PS format
Save graph as PDF	Save the graph in Adobe's (Acrobat) PDF format
Save graph as PNG	Save the graph in Portable
Save graph as WMF	Save the graph in Windows metafile format
Save graph as GIF	Save the graph in GIF (graphics interchange format).

These menu items will bring up a standard file selection dialog that can be used to locate files and provide the file name for open/save operations. The model parameters are saved in two files: the first one (\*.INP) defines files general properties and the second one (\*.BLX) defines the actual block model data. When saving the results, GRABLOX2 stores text information into a \*.OUT file. By default, gravity data are in a \*.DAT file whereas gravity gradient data are in a \*.GAT file. All data files are stored in (ASCII/ISO8859) text format. Binary block (\*.BLX) files can be used to handle very large models more efficiently. See chapter 7 and the BLOXER manual for more information about the block file format.

GRABLOX2 does not support direct printing. Instead, graphs need to be saved on a disk first and then a suitable third party program (e.g., Adobe Acrobat Reader, Ghostview, Xnview, etc.) can be used to print them or copy to clipboard. The graphs are saved in landscape A4

size as they appear on the screen. The GIF file is the only bitmap format (size 2970×2100 pixels). The Enhanced Postscript file does not include preview bitmap.

#### 4.2 Gravity menu

Comp. -> Meas.	Computed data is made as (synthetic) measured data
Comp. -> Regi.	Computed data is made as (constant) regional field
Subtract regional	Regional field is subtracted from measured data
Remove measured	Remove all information about measured data
Compute default	Create a default model based on measured data
-----	
Scale unit	Define dimensions in meters, kilometers, feet or miles
Gravity field	Compute total field or anomalous field
Computation	Compute $g_z$ , $g_z$ & $g_{zz}$ or $g_z$ & 6 gradients & $g_{uv}$
Background	Define background density mode
Depth weight	Define depth weighting mode
Smooth edges	Define smoothing mode (on model borders)
Gravity options	Set additional computational options.

- *Comp→Meas* and *Remove measured* are useful when testing the program and using it for educational purposes.
- *Comp→Regi* can be used to define a regional field using the current model. Alternatively, one can use the *Read regional data* menu item.
- *Subtract regional* will remove the current regional field (if defined) and/or base anomaly from the measured data. Removal of the regional trend reveals finer features in the mapped data. The resulting dataset can be saved to disk for future use with the *File/Save data* item.
- *Compute default* is used to create an initial model. It is useful after new measured data has been read in, because will give the super-block some reasonable coordinates based on the data. If the data are regularly gridded the model is also discretised automatically, i.e., the model is divided into minor-blocks that are positioned below the data points (z coordinates also). When dealing with irregularly sampled data the user should perform the discretization manually. Note that 2-D surface data and profile data are handled differently and profiles that are not aligned with coordinate axes are not supported.

- When GRABLOX2 reads in data it does not know the units of the coordinates and distances. The *Scale unit* item defines the actual volume of the block model, and hence, the correct amplitude of gravity effect. Therefore, after reading in the data the user should supply correct scale (meters, kilometers, or miles) before continuing with the modelling or interpretation.
- The computed gravity field ( $g_c$ ) is the sum of 1) the gravity effect of the model ( $g_m$ ), 2) the user defined base anomaly ( $g_b$ ) and 3) external regional field ( $g_r$ )

$$g_c = g_m + g_b + g_r . \quad (6)$$

If regional field ( $g_r$ ) has been defined the *Gravity field* item can be used to include and exclude the regional field to give either the total field or the anomalous field. The base anomaly ( $g_b$ ) is always added to the computed response.

- *Computation* defines the three modes of the gravity computation:
  1. The traditional gravity effect, i.e., the vertical component of the gravity field ( $G_z$ )
  2. The gravity effect and the vertical gravity gradient ( $g_z$  &  $g_{zz}$ )
  3. The gravity effect and the full gravity tensor, i.e., the six gradient components.

Note that this menu item cannot be used after measured data (gravity and gradient) has been read in – the data itself defines the components that are going to be computed.

- *Background* defines the three modes of background density:
  1. Normally, the background density has a constant value defined by the *Bg dens* field on the left control panel. In this case, the background density is subtracted from the density values of each minor block. Thus, if the density of the block is equal to the background value it has no effect in the computed gravity anomaly.
  2. In large-scale crustal studies the background density value can be defined as a function of depth using the mean density of each layer. In this case, the mean density is subtracted from the density of each minor block. This method is useful in large-scale studies and when the density is assumed to increase downwards.
  3. The background density can also be defined as the mean density of all the model blocks. This method allows a simple way to define floating background value. If the density increases downwards this method is not as good as the second one.

The purpose of the background density is to remove (or reduce) the effect of the four sides of the main block especially when the model deals with absolute density values. If the contrast between the mean density of the model and the background is large, the effects of the internal density variations of the model will not be visible at all.

If the background density is zero, the inversion will resolve residual density where the mean density of the modelled volume should be added to give geologically realistic density values. When absolute density values, for example, from petrophysical samples are used to define the initial model, the background value cannot be zero anymore. In that case, the background value should be set to the mean of the sample values or, as usual, to the density value used in Bouguer and topographic corrections (e.g., 2670 kg/m<sup>3</sup>).

*Depth weight* defines three modes of depth weighting used in density inversion. Normally, the near-surface blocks have much larger sensitivity than the deep blocks. Consequently, the inversion will generate a model where the masses are concentrated close to the surface, which is likely to be geologically unrealistic. Depth weighting will place the mass distributions at their "correct" depth by enhancing the importance and sensitivity of the blocks as a function of depth. The depth weighting is discussed more in chapter 5.3.

The *Gravity options* sub menu contains following items:

- *Autosave on/off* enables and disables saving the model and data between each inverse iteration. Since the inversion of large models is usually made overnight, this allows to see how the model evolves in the inversion and to start the inversion again from any previous model.
- *Roughness on/off* enables and disables the use of discontinuity information in the Occam inversion. The discontinuity information is stored in the model file as a second parameter with the help of the BLOXER program. If the model file does not contain discontinuity parameter, this menu item will be inactive. See chapter 5.5 for more information.
- *Laplacian on/off* imposes Laplace's condition on the diagonal elements of computed gravity gradients,  $g_{xx}+g_{yy}+g_{zz}=0$ . This option is used mainly for testing purposes.
- *Inversion on/off* enables and disables inversion and defines whether or not the sensitivity matrix (Jacobian) will be computed automatically during forward computation when data have been read in. For example, when *Comp→Meas* item is used to set computed

(synthetic) data to measured data, inversion (and related menus and controls) is not enabled until it is manually enabled with this menu item. Note that when actual measured data is read in, the inversion is enabled by default. However, if the model is very large the program may be unable to allocate memory for the Jacobian. In this case GRABLOX2 disables the inversion and computes only the forward solution. After a new model with smaller amount of elements has been defined this menu item can be used to enable the inversion again.

- *Gradient weights* item provides a quick way to set the global weights for each gravity gradient (tensor) component. Alternatively, one needs to edit the *G-scale* parameter for each gradient component manually.
- *Rotate gradient data* is used to rotate the measured gravity gradient data horizontally, i.e., to perform tensor rotation around vertical z-axis. If the measurement lines are not aligned with the geographical coordinate axes, it is advantageous to rotate the data coordinates so that the model will be aligned with the data. However, airborne gravity gradient data (e.g., AirFTG<sup>®</sup>) is normally aligned with geographical coordinate axes and, therefore, tensor rotation must be performed to adjust the tensor data accordingly. Note that tensor rotation is different from coordinate rotation and that the vertical gradient ( $g_{zz}$ ) is not affected by the horizontal rotation. Also note that currently GRABLOX2 does not support rotation of data coordinates – that must be made using some third-party (spreadsheet) program.

#### 4.3 Edit menu

Block type	Use either equal-height or increasing height blocks
Min/Max values	Define min & max parameter value (and colour scale)
Layer-wise fix/free	Set fix/free/hidden status of the blocks per layer
Layer-wise reset	Set the density of the blocks per layer
Add margins	Add margin blocks around the main block
Del margins	Remove the outermost margin area
-----	
Labels	Define parameter labels shown in the graphs
Color scale	Change the colour scale
View options	Set view related settings.

- Although the *xyz* dimensions of the blocks can vary, all blocks have equal size by default. In the alternative block type (*Block type 2*) the height of the blocks will systematically



increase downwards by the value of the topmost layer of blocks. For example, layer thickness increases as 100, 200, 300, 400, 500 m giving depths 0, 100, 300, 600, 1000, 1500 m. The alternative block sizing thus reduces the resolution at the bottom but helps decreasing the total number of blocks. Note that block type cannot be changed if the *Shift* only editing mode is active (the second item in left control panel).

- *Min/Max values* item changes the limiting density values on the colour scale (below the layers or sections view) and determines the absolute minimum and maximum density values used in the inversion. These values also determine the maximum parameter steps in SVD based inversion.

*Note about the input dialogs:* The parameter values are passed to the program via a text string using a simple input dialog that appears on the display. The dialog does not contain normal Cancel button. Therefore, to cancel the input operation one should close the dialog with force (pressing the cross icon at the top-right corner) or provide an empty input string. This rule applies to all other similar input dialogs.

- *Layer-wise fix/free* option allows setting the fix/free status (weights) and the visible/hidden status of the blocks of each layer manually. Weights have integer values that range between -101 and 100. Zero weight will fix the density of the block during inversion. Weight equal to 1 and 100 make the density fully free and values ranging between 2 and 99 provide decreasing weight so that 2 is almost fully fixed and 99 is almost fully free. Negative weight value will set the block hidden (the block will still carry the value of the weight). Note that computationally, hidden block has density that is equal to the background density. Hidden blocks are not drawn in layer and section views. See chapter 5.2 for more information about the fix/free status.
- *Layer-wise reset* option allows setting the density value of the blocks for each layer manually. Fixed and hidden blocks are not affected.
- *Add margins & Del margins* can be used to add a single row/column of blocks horizontally around the whole block model at once. Margins are important when interpreting gravity gradient data, because they can be used to move the edge effects (due to the density contrast between the block and the background) outside from the computation area. When adding margins an input dialog appears and the user is asked to provide the width of the margin blocks.

- *Labels* either removes the layer and section labels totally, or shows the numeric value of the density, block height, block depth, fix/free status (weights), or the discontinuity (if available).
- *Color scale* item is used to choose between normal rainbow scale, inverted rainbow scale, normal grayscale, inverted grayscale and a special (seismic/temperature) red-white scale.

The *Edit options* submenu contains following items:

- *Show/Hide grid* can be used to clarify the contour/image maps as well as the layer and section graphs.
- *Contour/Image* item swaps the display mode of gravity data between a contour map and an image map where data are plotted as coloured rectangles. Contour plots are smoother but more time consuming if there are lots of data points. Highly irregularly spaced data should be as an image map because the interpolation can create artifacts.
- *Show/Hide regional* item enables/disables plotting the regional field or base anomaly. When the regional field is not shown one can swap between computed and measured data map to see the visual difference between them more easily.
- *Show/Hide difference* enables/disables plotting the difference between computed and measured data as a map. When the difference is not shown one can swap between computed and measured data map to better see the visual difference between them.
- *Swap map area* changes the limits of the data and layer maps so that they are either based on the model area or the data area. If wide margins have been added to the model, this option allows the user to focus either on the model area or the data area.
- *Difference abs/%* item defines the difference between the computed and measured data either as absolute difference (data units) or as relative error (in percents). The relative error is computed with respect to the (max-min) variation of the measured data.

#### 4.4 Knots menu

Knots are a set of manually defined gravity points which can be used to represent the regional field. This representation is made either by 2D interpolation or the base anomaly is optimised on the knots instead of the gravity data. Knots are not used with gravity gradient data. Knots are created and edited interactively on the profile response view. Their location can be edited interactively on map views as well, but their level (gravity value) is defined by given value. Knots are shown as crosses and circles on profile views and data maps, respectively.

Add knots	Add knots on the current profile section
Edit knots	Edit knots on the current profile
Delete knots	Delete knots from the current profile
-----	
Knots->regional	Interpolate a regional field through the knot points
Regional->knots	Shift the knots on the level of the current regional field

The interpolation of the regional field (*Knots->Regional*) is made using a simple inverse distance weighting. The idea is to add few sparsely spaced knots (minimum is 5 knot points) on few sparsely spaced profile lines. These knots define a set of points through which the regional field is then interpolated. The interpolated regional field replaces any separately defined regional field read from a file. The base anomaly (defined by the coefficients of a second degree polynomial) is still added to the computed regional field as usual.

If a user defined regional field and/or a base anomaly already exists, the *Regional->knots* option allows shifting all current knot points on the level of the regional field. Again, a simple inverse distance weighting scheme is used. This option is useful after the knot locations have been added on the map view, because it provides a method to give the knots good initial values that can then be edited in profile views.

Adding and editing of knots is also possible using the buttons at the bottom of the left control panel of the GUI window. Please, see chapter 6.6 for more information on the knots.

#### 4.5 Exit menu

*Widescreen* item is used to restart the GUI so that it fits the (old) 4:3 displays or (modern) widescreen and laptop displays better. When entering widescreen mode the user needs to provide a value for width/height ratio that is normally between 0.7 and 0.8.

*Exit* item is used to confirm the exit operation. On exit the user is given an opportunity to save the current model and the results, provided that exit is made without an error condition. On exit, the program first asks the name for the model files (\*.INP & \*.BLX) and then the name of the results file (\*.OUT). If the user cancels the save operation after choosing to save the model or results, the default filename GRABLOX2.\* will be used.

Errors that are encountered before the GUI starts up are reported in the GRABLOX2.ERR file. When operating in GUI mode, run-time errors arising from illegal parameter values are displayed on the screen. Some errors, however, cannot be traced and given an explanation. The most typical problem is perhaps a corrupted GRABLOX2.DIS file that prevents the program to start up properly. This problem can be circumvented simply by deleting the \*.DIS file. A missing or incorrectly named \*.BLX file can also cause troubles. Please, note that most messages that appear on the console window are stored also into a run-time log file GRABLOX2.LOG. This file is written over when the program is restarted.

#### 4.5 Left control panel

*Update* button is used (and must be used) to validate the changes made to the parameter text fields. Once the *Update* button is pressed, the parameters defining the position, size, and discretization of the block model are read in, checked for erroneous values and the old values are replaced with the new ones.

The use of *Update* button invalidates existing computed results and a new forward computation is needed.

The list widget below the *Update* button defines how the block parameters will be affected when the *Update* button is pressed.

1. *Shift only* mode updates only the *xyz* position of the whole block model. The size and the discretization of the main block cannot be changed in *Shift only* mode. This is the default mode and its purpose is to protect the model from accidental changes.
2. *Ignore* mode allows easy redefining of the size and discretization of the main block. As the name implies, the existing relationship between the position and the density of the blocks is ignored totally. The density of an arbitrary block  $p(i,j,k)$  remains the same but it will most likely move to a totally different position inside the model if the discretization (indexing) changes.

3. *Preserve* mode uses simple 3-D nearest neighbour interpolation to define new density values of the blocks when the model is resized, re-discretised, or repositioned. For example, if the position of the main block is shifted, the anomalous target remains more or less at the same geographic position, but its relative position (indexing) inside the main block will change. Note that 3-D interpolation of densely discretised and large models can be very time-consuming.

The following text fields define the position, size and discretization of the main block:

1. *X* position (easting) of the SW corner of the main block
2. *Y* position (northing) of the SW corner of the block
3. *Z* position (depth to the top) of the main block
4. *X* size of the main block (in EW direction)
5. *Y* size of the main block (in NS direction)
6. *Z* size or height of the main block (downwards)
7. *X* discretization (number of blocks) in EW-direction
8. *Y* discretization (number of blocks) in NS-direction
9. *Z* discretization (number of blocks) in vertical direction.

In GRABLOX2 the true dimensions (distances) of the block model (and computation grid) are defined using the *Gravity/Scale unit* menu item. The given rectangular

coordinates do not need to be actual geographic map coordinates. To be able to better model geological strike directions, for example, the data and the model should be defined using rotated coordinates.

The next two text fields define two density parameters:

1. *Bg dens* defines the user defined background density value,  $\rho_0$  (g/cm<sup>3</sup>)
2. *Param* defines the default density value used to reset the model into,  $\rho_1$  (g/cm<sup>3</sup>).

The screenshot shows the GRABLOX2 software interface. The 'Update' panel is active, displaying various parameters for the block model. The 'Block model' dropdown is set to 'Shift only'. The 'X-posit' field is 3375, 'Y-posit' is 7487, 'Z-posit' is 0, 'X-size' is 102, 'Y-size' is 90, 'Z-size' is 30, 'X-divis' is 34, 'Y-divis' is 30, and 'Z-divis' is 8. The 'Density' section shows 'Bg dens.' as 2.67 and 'Param.' as 2.67. Below the 'Update' panel is the 'Reset dens.' panel, which includes a 'Data area' section with 'X-step' (3), 'Y-step' (3), 'X-start' (3520650), 'Y-start' (7448700), 'X-end' (3561810), 'Y-end' (7489390), and 'Z-level' (0). The 'Base' section shows the formula 'Base: A+Bx+Cy+Dx^2+Ey^2+Fxy', a 'Base A' dropdown set to 10, and a 'Value' field set to 10. At the bottom, there are buttons for 'Update base', 'Reset base', 'Add knots', and 'Edit knots'. A checkbox for 'Optimize base to knots' is also present.

Note about density units: In GRABLOX2 the density is defined in units  $[\text{g/cm}^3]$ . Use the BLOXER program to modify density values if required. For example, divide by 1000 (actually, multiply with 0,001) if densities are given as  $\text{kg/m}^3$ .

*Reset dens* button resets the density of the whole model to the value defined in *Param* text field. Note that fixed and hidden blocks are protected and will not be affected.

The gravity field  $[\text{mgal}]$  and gravity gradients  $[\text{Eo}]$  are always computed using some density contrast  $\Delta\rho = \rho_i - \rho_0$   $[\text{g/cm}^3]$ , that is to say, the difference between the density of the blocks ( $\rho_i$ ) and the background density ( $\rho_0$ ). The method used to define the background density is chosen using the *Gravity/Background* menu item. When *Given value* mode is active, the value of background density provided in *Bg dens* field will be subtracted from the density values of each minor block. Note that density values can be absolute (background density and density range is based on petrophysical data, for example,  $\rho_0 = 2,7 \text{ g/cm}^3$  and  $2,0 < \rho < 3,6 \text{ g/cm}^3$ ) or relative (background value is zero or arbitrary, e.g.  $-0,6 < \rho < +1,2 \text{ g/cm}^3$ ). The computed gravity results will be exactly the same if the density contrasts with respect to the value of background value are the same.

In forward computations the rectangular computation grid is defined by the seven text fields below *Reset dens* button:

1. *X-step*: spatial step between computation points in  $x$  direction,  $dx$
2. *Y-step*: spatial step between computation points in  $y$  direction,  $dy$
3. *X-start*:  $x$  coordinate (easting) of the grid start position,  $x1$
4. *Y-start*:  $y$  coordinate (northing) of the grid start position,  $y1$
5. *X-end*:  $x$  coordinate (easting) of the grid end position,  $x2$
6. *Y-end*:  $y$  coordinate (northing) of the grid end position,  $y2$
7. *Z-level*: constant  $z$  level (height or elevation) of the computation grid,  $zh$ .

If the width of the computation grid is zero (e.g.,  $x1 = x2$  or  $y1 = y2$ ) the computations are made along a single profile that extends from  $(x1, y1)$  to  $(x2, y2)$  and *X-step* is used as a distance step between the computation points along the profile. In this case the profile is aligned either with  $x$  axis ( $y1 = y2$ ) or  $y$  axis ( $x1 = x2$ ). A more general (azimuthal) profile from  $(x1, y1)$  to  $(x2, y2)$  can be defined, if *Y-step* is set equal to zero ( $dy = 0$ ). In this case the *X-step* is used as a true distance between the points along the slanted profile. Naturally,

profile data can be plotted only as a profile plot (no contour or image maps). However, in this case the data are drawn exactly without any interpolations. Also note that depending on the current status of data extent (defined by *Edit/Edit options/Swap map area* menu item) the layer view shows only the part of the model below the profile.

Note about Z-level and X- and Y-steps: The abovementioned values are defined separately for gravity and gravity gradient data. The *Gravity/Gradient* button in the right control panel is used to swap the currently active data component. The information text in the graph shows the current data mode.

Measured data are assumed to be irregularly spaced. The computations are always made at the actual data locations  $(x,y,z)$ . Therefore, grid start and end positions are inactive after measured data have been read in. Moreover, when data has been read in *X-step* and *Y-step* define grid spacing used in contour and image maps. When the data are irregularly spaced, x and y steps should be edited manually to reduce possible contouring artifacts.

Note about data elevation: The z coordinates (elevation or topography) of the measured data are used only if they are provided with the data AND the *Z-level* is equal to the top of the main block ( $Z\text{-level} = Z\text{-posit.}$ ). Otherwise, the response will be computed at the constant elevation defined by the *Z-level*. This method allows inspecting the effect of height in gravity computations. To compute the gravity field on the surface of the block model, t must set the *Z-level* very close to the *Z-posit* value (but do not make them equal).

The pull-down list widget and the text field *Value* below it are used to define the coefficients of a second degree polynomial used for computational base anomaly of a form:

$$g_b(x,y) = A + B(x-x_0) + C(y-y_0) + D(x-x_0)^2 + E(y-y_0)^2 + F(x-x_0) \times (y-y_0) , \quad (7)$$

where  $x_0$  and  $y_0$  define the shifted origin for the base function. By default the origin  $(x_0, y_0)$  is located at the middle of the main block. Because the amplitude changes of gravity anomalies are small compared to the distances, the values of  $C$  and  $D$  are scaled (multiplied) by 1000 and the values of  $E$  and  $F$  are scaled by million. Thus, if dimensions are defined in meters, the linear and second order coefficients are defined as mgal/km and mgal/km<sup>2</sup>, respectively. The base anomaly is always added to the computed gravity anomaly even if external regional field has been read in.

Note about the base anomaly: The base anomaly is defined separately for the gravity data ( $g_z$ ) and each gravity gradient component ( $g_{xx}$ ,  $g_{xy}$ ,  $g_{xz}$ ,  $g_{yy}$ ,  $g_{yz}$ ,  $g_{zz}$  and  $g_{zz}$ ) depending on the current active data component defined by *Gravity/Gradient* button. In other words, when gravity data is active (is currently mapped or plotted) the coefficients are related to the base anomaly of the gravity data and when gradient data is active the coefficients are related to that.

*Update base* button at the bottom of the left control panel is used to update the changes made to the base level immediately without the need to do full forward computation. *Reset base* button is used to reset (null) the first and second order coefficients of the base anomaly of the currently active data component. In other words, *Reset base* resets all other coefficients but the level ( $A$ ).

*Optimize base to knots* check box changes the way base anomaly inversion is made is inactive if there are no knot points. Knots are used to define a user defined regional field by 2D interpolation. Knots can also be used to define a set of gravity data points upon which the base anomaly will be optimised. *Add knots* and *Edit knots* buttons enable interactive editing mode where knots can be inserted into map and profile views. The level of existing knots can be edited only in profile view.

#### 4.6 Right control panel

The *Compute* button is used to perform the forward computation using the current values of the model and system parameters. The *Optimize* push button is used to start the inversion. Note that depending on the total number of minor blocks and the amount of computation points, the inversion can take a considerable amount of time.

The list widget below the *Optimize* button defines the three inversion modes:

1. *Base* optimizes the coefficients of the base anomaly,
2. *Density* optimizes the density of the free minor blocks using SVD based method
3. *Occam density* optimizes the density of the free minor blocks using Occam's method, where both the data misfit (difference between the measured data and the computed response) and the model roughness are minimised.

The second list widget defines which coefficients or a combination of coefficients of the base anomaly is going to be optimised when *Base* optimisation is performed. Note that in *Occam density* inversion the linear coefficients ( $A$ ,  $B$ , or  $C$  or  $A$ ,  $B$  and  $C$ ) of the base anomaly of the



gravity data (not the gradient data) can be optimised together with density values. One must remember to select *None* if one does not want the base anomaly to be optimised together with density. Note that in practice combined inversion of base anomaly and block parameters requires that some density values are fixed based on additional *a priori* information.

The optimization of the base anomaly is performed either on the gravity data or gravity gradient data depending on the current status of the mapped data defined by the *Gravity/Gradient* button.

The following five text fields define some inversion related options. See chapter 6 for more information about the abovementioned inversion options.

1. *Iters* defines the number of iterations made during one inversion run.
2. *Thres %* defines the strength of damping in SVD based inversion. Normally, the threshold should be at the level of the smallest singular values that are still informative. The smaller the threshold value is the weaker is the damping. Note that the value is actually multiplied by 100. Thus, the default value 0.01 (%) is actually 0.0001, and all singular values  $< \lambda_{\max}/10^5$ , where  $\lambda_{\max}$  is the maximum singular value, will be damped.
3. *F-option* provides a numerical (real) value for an additional parameter to the inversion. Its role depends on the selected inversion mode. In SVD based inversion it determines the maximum step controlling the convergence. In Occam inversion it is used as a Lagrange multiplier or scaler controlling data fit vs. model smoothness.
4. *I-option* provides a numerical (integer) value for an additional parameter to the inversion. Its role depends on the inversion mode – in Occam inversion it is used to control the rigidity of fixed blocks whereas in base

inversion it is used to shift the fit towards the bottom or the top of the gravity or gradient anomaly.

5. *W-norm* increases or decreases the depth weighting as a function of depth. The value should be changed from the default value 1,0 only if the inversion results seem to overestimate or underestimate the density values at the deep parts of the model. For example, *W-norm* value 2,0 enforces twice as strong depth weight at the bottom of the model and a value 0,5 gives only half as strong depth weighting.
6. *G-scale* defines global data weight for individual data components when gravity data and gradient data are optimised together. Depending on the current status of the mapped data (*Gravity/Gradient* button) the weight is related to gravity data or one of the gravity gradient components. The value should be changed from the default value only if the inversion seems to emphasize or de-emphasize one of the data components too much. For example, decreasing the global weight of  $G_{xy}$  gradient component to 0,1 will make the corresponding data error 10 times smaller and, consequently, gives that data component less importance in the inversion.

The next three push buttons are used to show and define the current gravity data.

- *Map data* button shows the gravity data as a coloured 2-D map. If measured data has been defined, repetitive use of the *Map data* button will swap the view between the computed and measured data, the data error (difference between measured and computed data) and the regional or base response data. The map type can be changed between contour or image map using *Contour/Image* item in the *Edit/Options* submenu. Plotting of data error and regional field can be omitted using *Show/Hide difference* and *Show/Hide regional* items in the *Edit/Options* submenu.
- *Gravity/Gradient* button swaps the current data component between gravity field and gravity gradient. The item is active only if gradient computation has been enabled (using *Gravity/Computation* menu item).
- *Grad comp.* button swaps the current data between different gradient components when full gravity tensor data is being computed. The item is active only if gradient computation has been enabled (using *Gravity/Computation* menu item).

*Layr/Sexn* text field shows the index number of the current layer or section when the model is being visualised (see the meaning of the buttons below). It is possible to jump from one layer

or section to another by first giving a new value for the index number manually and then pressing the *Layers* or *Sections* button.

- *Layers* button is used to show the density values across a horizontal layer.
- *Sections* button is used to show the density values across a vertical cross-section.
- *Profiles* button is used to show the computed and measured data and the regional/base anomaly as XY graphs over the data area. The data is either gravity or gravity gradient depending on the status of current map defined by the *Gravity/Gradient* button.
- *Crossing dir* button is used to change the direction of the sections views and data profiles between (vertical) SN and (horizontal) WE directions.
- *<-/->* button is used to change the rotation direction of contours/image maps, layers, sections and profiles when the corresponding buttons are pressed.

When the *Layers*, *Sections*, and *Profiles* buttons are pressed multiple times the current layer, section or profile is rotated, that is to say it changes to the next or previous one depending on the direction defined *<-/->* button. After the last (or first) layer, section or profile the view jumps back to the first (or last) one. One can jump directly into a desired layer or section by first providing the index number on the *Layer/Sexn* text field and then pressing the corresponding push button.

Normally, section view shows only a vertical cross-section of the model and profile view shows the profile response. Activating the *Profile+Section* changes the graphs so that the profile response is shown on top of the model cross-section. In this case, pressing the *Section* or *Profile* buttons will continue swapping the profile or the section while the other view (section or profile) remains the same. The *Align prof/section* button is used to align the profile and section views. If profile view was changed the index number of the section will be changed so that it will show the model below the profile. Likewise, if section view was changed the profile closest to the section will be shown in the upper graph.

If the data covers a two-dimensional area it is assumed to be irregularly sampled and the profile graphs show the data interpolated along (imaginary) *x* or *y* directed profiles, the interval of which are based on values of the *X-step* and *Y-step* text fields in the left control panel. This means that the profile response displays the original data only if the data are regularly gridded and the steps are equal to that of the data. If the data are highly irregular, the DISLIN

contouring algorithm used in GRABLOX2 may not work well and the profile response shows up incorrectly. Some improvement can be made by changing the data representation from a contour map to an image map using the *Contour/Image* item in *Edit/Options* submenu. Alternatively, the data could be mapped on a regular grid beforehand using some third party interpolation software (e.g. Golden Software Surfer or Geosoft Oasis Montaj).

GRABLOX2 suits also 2.5-D interpretation of single profile data. In this case, the data should be aligned with either the x or y axis so that the *Gravity/Compute default* can be used to generate a proper initial model automatically. Consequently, if the profile is aligned with y axis, the model will be discretised along the x and z directions, the y discretization (*Y-divis.*) is set equal to one, and the strike length (*Y-size*) is made so large that the model appears to be two-dimensional. Although it is possible to compute the forward response along general profiles that are not in x or y direction, a 2.5D model cannot be constructed properly, since the model is always discretised along x and y axes. Note that 2.5-D inversion methodology has not been tested thoroughly. It is likely that the depth weighting needs to be redefined for models having elongated strike length.

Normally, the vertical depth axis is plotted in 1:1 scale with horizontal axes in X and Y-section views. *Vert. scale* slide control can be used to stretch or squeeze the depth axis. It is useful if the height of the model is considerably smaller than its horizontal dimensions. The other three slide controls (*Horiz. rotation*, *Vert. rotation* and *Rel. distance*) are used to change the view point of the small 3-D model view next the main graph.

The *Update graph* button is used to validate the changes made to the slide controls and to redraw the current graph.

## 5. Inversion modes

The three inversion modes are:

1. *Base* SVD based optimisation of the base anomaly (coefficients *A-E*)
2. *Density* SVD based (unconstrained) optimization of the density
3. *Occam* (constrained) Occam inversion of the density.

Density values are limited by the minimum (*Min*) and maximum (*Max*) values of the colour scale below the *Layer* and *Section* graphs. The limit values are set using the *Min/Max values* item in the *Edit* menu.

### 5.1 Regional field

The measured gravity field is always affected by mass distributions both near to and far away from the study area. The purpose of the regional field in GRABLOX2 is to account for (and subtract) the gravity effect of all those masses that are located around and below the volume of the 3D super-block. Although the regional field can be defined almost arbitrarily, the important thing is not to remove any gravity effects that might be explained by density variations inside the volume of the 3D block model.

The three ways to define the regional field in GRABLOX2 are:

1. User-defined base anomaly is defined by a second degree polynomial,
2. Regional field is read in together with the measured data (*Read data*), or
3. Regional field is read in from a separate file (*Read regional*).

The choice of the regional field is one of the most important factors of successful gravity interpretation. Best results are obtained if the regional field is interpreted separately using a large regional model and gravity data that extends 2–5 times farther than the actual interpretation model of the study area. The block size of the regional model can be 2–10 times larger (horizontally and vertically) than what will be used in local modelling. The discretization of the local and regional models should be set so that the block boundaries (sides and bottom) of the local model fit the discretization of the regional model. Once the regional-scale inversion has been made and an acceptable model has been found, the volume of the local study area is removed (hidden) from the regional model (with BLOXER) and the regional, large-scale gravity anomalies of the surroundings are computed at the data locations of the local study area and saved into file. Alternatively, surface fitting or low-pass filtering

(e.g., Surfer, Geosoft, Fourpot) can be used to define the regional trend of the gravity anomaly. These methods, however, are somewhat vague, because they do not take into account the real physics and the depth extent of the targets.

The regional trend of the gravity data can be defined using the optimization of the parameters of the base anomaly. Although a second degree polynomial used in GRABLOX2 cannot describe as complicated regional effects as external files, it is often a good approximation to the regional gravity effects. Also note that the base anomaly is useful also if the data and/or regional field for some reason are not in level with the measured gravity data.

Note about the base anomaly: Base anomaly is always added to the computed response. Nonetheless, it can be used together with the user defined regional field also. Therefore, optimization of the coefficients of the base anomaly provides a method to adjust the first and second degree behaviour of the regional field. Furthermore, in *Occam* inversion the base level and/or linear trend ( $A+Bx+Cy$ ) of the base anomaly of the gravity field can be optimised together with the density values (although this can be ambiguous).

The *Gravity/Subtract regional* item can be used to remove the regional field from the measured data and the new data can be saved into a file (*File/Save data*). Theoretically, there should not be any difference if the inversion is made using total field or anomalous field. However, if the regional trend has been removed from the original data, the visual inspection of the data and fit between measured and computed data is often much easier. Moreover, the absolute minimum and maximum values and the dynamic range of the data will be different. As a consequence, inversion uses different data weighting and is likely to yield slightly different results. The users are advised to work with anomalous field if a very strong regional trend is included into the original gravity data.

## 5.2 Fix/free status

The fix/free status defines whether the block is included into the inversion (optimization) or not. In GRABLOX2 it serves also as weight factor defining relative importance of the fixed density values. The weight factor is utilised only in Occam inversion. The fix/free value is zero (0) for a totally fixed block (not optimised at all), and either one (1) or one hundred (100) for a totally free block. Fix/free values ranging from 2 to 99 define variable weights. Note that in a \*.BLX file, the fix/free status is stored as an integer value.

The *Layerwise fix/free* item can be used to manually set a constant fix/free value for each layer. Note that it can also be used to set the hidden/visible status of the blocks per each layer. Hidden blocks are removed totally from the computation. As such they represent the density of the background medium. For more advanced editing of the fix/free and hidden/visible status one should use the BLOXER program.

### 5.3 Depth weighting

Normally unconstrained (and un-weighted) gravity inversion tends to put the excess mass mainly in (or remove the missing mass mainly from) the uppermost blocks, because their sensitivity is much greater than that of the deeper blocks. Consequently, the resulting inverse model is not likely to be geologically realistic. The concept of depth weighting (Li and Oldenburg, 1996 and 1998) tries to overcome this problem. Basically the idea is to multiply the elements of the sensitivity matrix and the difference vector with depth dependent function of form

$$w_d = (z + z_0)^a, \quad (8)$$

where  $z$  is the depth of a block from the top of the main block,  $z_0$  is a scaling value that depends on the size of the blocks, and  $a$  is an exponent that depends on the geometric form of the potential field. The gravity field has  $1/r^2$  dependency on the distance from the source. However, experimental value  $a = 1$  gives better convergence in 3-D gravity inversion.

One problem with depth weighting is that it does not take *a priori* data into account and possible near surface mass distributions will get incorrectly positioned. To circumvent these problems GRABLOX2 has an option to automatically relax the depth weighting based on the value of the current RMS error. When the RMS error decreases during the inversion so does the depth weighting until it disappears when RMS reaches level of 0,2 %. In practice, however, full depth weighting should be used and the other two options (relaxed or discarded) should be used just to test the effect of depth weighting.

The status of the current depth weighting method can be derived from the last line of the information text next to the model and response graphs. After the inversion mode (*Base optimization, SVD inversion, Occam inversion*) the first integer number defines the depth weighting mode (0= none, 1= relaxed, 2= full). The second number shows whether or not the

automatic model saving is active or not (0= no, 1= yes). If the model file includes discontinuity information the third number would indicate if it is active or not (0= no, 1= yes).

#### 5.4 Normal SVD based inversion

*Base* and *Density* inversion modes are based on a linearised inversion scheme similar to that defined in Jupp and Vozoff (1975) and Hohmann and Raiche (1988). The optimization uses singular value decomposition (SVD) and adaptive damping method defined in Pirttijärvi (2003). For the sake of clarity, the basics of the linearised inversion method are discussed in the following.

The objective of the numerical inversion is to minimize the difference or misfit ( $\mathbf{e} = \mathbf{d} - \mathbf{y}$ ) between the measured data,  $\mathbf{d} = (d_1, d_2, \dots, d_M)$  and the computed response  $\mathbf{y} = (y_1, y_2, \dots, y_M)$ . Assuming an initial model with parameters  $\mathbf{p} = \mathbf{p}_0 = (p_1, p_2, \dots, p_N)$ , forward computation is made to yield model vector  $\mathbf{y}$  and the sensitivity or Jacobian matrix,  $A$ , the elements of which are partial derivatives of the model data with respect to model parameters ( $A_{ij} = \partial y_i / \partial p_j$ ). Thus, the linearization of the problem leads to matrix equation

$$A \cdot \Delta \mathbf{p} = \mathbf{e} . \quad (9)$$

Solution to Eq. (9) gives parameter update vector  $\Delta \mathbf{p}$ , which is then used to give updated parameter vector,  $\mathbf{p}_1 = \mathbf{p}_0 + \Delta \mathbf{p}$ . The process is continued iteratively until desired number of iterations has been made and/or the fit between the measured and computed data becomes small enough. The root-mean-square (RMS) error is used as the measure of the data fit

$$RMS_d = \sqrt{\frac{1}{M} \sum_{i=1}^M \left( \frac{d_i - y_i}{\Delta d} \right)^2} , \quad (10)$$

where  $\Delta d = \max(\mathbf{d}) - \min(\mathbf{d})$  is used to scale the data. The singular value decomposition (SVD) of the Jacobian is (Press et al, 1988)

$$A = U \Lambda V^T, \quad (11)$$

where  $U$  and  $V$  are matrices that contain the data eigenvectors and data eigenvectors ( $T$  means transpose operation) and  $\Lambda$  is a diagonal matrix that contains the singular values. Since both  $U$  and  $V$  are orthogonal matrices, the solution to Eq. (9) is



$$\Delta \mathbf{p} = V \mathbf{A}^{-1} U^T, \quad (12)$$

where the elements of the diagonal  $\mathbf{A}^{-1}$  matrix are simply  $1/\lambda_i$ . If some of the singular values are very small the inverse problem becomes ill-posed. Therefore, stabilization or damping is performed by adding a small value, so-called Marquart factor,  $\beta$ , to the singular values giving  $1/(\lambda_i + \beta)$ . In practice, the singular values are multiplied with damping factors (Hohmann and Raiche, 1988)

$$t_i = \begin{cases} \frac{s_i^4}{s_i^4 + \mu^4} & s_i \geq \mu^2 \\ 0 & s_i < \mu^2 \end{cases}, \quad (13)$$

where  $s_i = \lambda_i / \max(\lambda)$  are normalised singular values and  $\mu$  is the relative singular value threshold. Thus, elements of  $\mathbf{A}^{-1}$  that correspond to very small singular values are removed totally and the remaining values are damped so that they are never too small. The damping diminishes the effect of small singular values and the threshold is used to control the strength of damping.

*Thres %* field defines the minimum value of the relative singular value threshold. Normally, it should be at the level of the smallest singular values that are still "informative". The threshold should be changed only if the inversion becomes unstable. Larger threshold value forces stronger damping, which suppresses the effect of small singular values and improves the stability of the inversion. Note that the %-character is used just to remind that the value is actually multiplied by 100. Thus, the default value 0.01 is actually  $10^{-4}$ .

*F-option* parameter defines the maximum parameter step for the adaptive damping (Pirttijärvi, 2003). Small value of *F-option* makes parameter steps smaller, increases the value of relative singular value threshold and thus the damping, and (usually) makes the convergence slower. Large value of *F-option* makes parameter steps bigger, reduces damping closer to that defined by minimum singular value threshold, and thus increases the convergence. However, it may also lead to unstable convergence. In practice, the *F-option* value should range between 10–0,1 and the default value 1,0 provides relatively good convergence.

The numerical computation of the SVD is the most time-consuming part of the inversion. It has approximately  $O^3$  dependence on the total number of minor blocks. Because SVD based inversion can be time consuming, Occam methods that use iterative (conjugate gradients or QR) solvers are preferred when the number of blocks is  $> 10000$ .

### 5.5 Occam inversion

The famous citation from Wilhelm of Occam (Ockham) from the 14.th century states: "*Non sunt multiplicanda entia praeter necessitatem*", which translates "Plurality should not be assumed without necessity" or "One should not complicate matters beyond what is necessary". This so-called "Occam's razor" means that if two solutions are equally good the one that is simpler should be preferred. In geophysical inversion practice Occam's method means that in addition to minimizing the fit between the measured and computed data the roughness of the model should be minimised also. In practice, Occam inversion will produce smoother models because the neighbouring density values are used as constraints. As a drawback, the models may not fit the data as well as the unconstrained (SVD) inversion. However, the so-called Lagrange multiplier (provided via *F-option* parameter) can be used to control the minimization of either the data error (fit) or model roughness (smoothness).

*Occam density* inversion mode optimizes the density of the free blocks. An important feature of Occam inversion in practice is that if the densities of some blocks are fixed, Occam's method will constrain the parameters of the surrounding blocks to fit those of the fixed blocks. For example, petrophysical data can be used to constrain the density at the surface.

The ability to constrain the parameter values reveals another advantage of Occam's method. The Occam inversion can create a smooth and continuous model, even if the measured data are irregularly spaced and do not cover the whole model area. In such situations the normal (unconstrained) SVD based inversion gets unstable and tends to generate very rugged (chess-board) structures.

The roughness,  $\mathbf{r}$ , is defined as the difference between the density of a minor block from the mean density of the surrounding blocks,  $\mathbf{r} = \mathbf{p} - \mathbf{p}^*$ . In practice, the roughness (or its reciprocal, the smoothness) is incorporated into the Eq. (9) by adding the sensitivities of the roughness ( $B_{ij} = \partial r_i / \partial p_j$ ) and extending the data error vector with the model error  $\mathbf{e}'$ , which in general is computed against some reference model, but in this case the null-model so that  $\mathbf{e}' = \mathbf{r}$ . Occam's method leads to a new matrix equation

$$(A + \alpha B) \cdot \Delta \mathbf{p} = (\mathbf{e} + \mathbf{e}') . \quad (14)$$

where  $\alpha$  is so-called Lagrange scaler (multiplier) that is used to emphasize either the fit between the data or the smoothness of the model.

Since the number of data  $M$  is typically equal (or smaller) than the number of blocks in the top layer, the dimension of the linear system increases quite drastically,  $N \times M \rightarrow (N \times (M+N))$ . In this case the SVD would become too slow to be practical. Therefore, Occam inversion uses either iterative conjugate gradient (CG) algorithm or QR decomposition (Householder rotations) to solve the matrix system for the parameter steps. The CG solver is faster than QR but it requires an additional matrix for the normal equations ( $A^T A$ ). Therefore, CG is used as a default solver, but QR method is automatically chosen if memory cannot be allocated for the normal equations.

In *Occam density* inversion the coefficients of the linear part of base anomaly ( $A+Bx+Cy$ ) can be optimised together with density values if the second scale widget in the right control panel is set to appropriately (*Base A*, *X-grad B*, *Y-grad C* or *Linear ABC*). The second order coefficients cannot be optimised because they would most probably remove (or add) excess mass from the gravity anomaly itself.

*F-option* defines the Lagrange scaler ( $L = \alpha$ ) in the constrained *Occam density* inversion. Lagrange scaler controls if the inversion should give more importance to minimizing the data error instead of minimising the model roughness, or vice versa. Since both the data values and the model parameters are normalised, the default value ( $L= 1.0$ ), usually gives approximately the same importance to the data and the model error. Increasing the Lagrange scaler ( $L > 1.0$ ) will emphasize the importance of model roughness and the inversion will create smoother models with the expense of increased data misfit. Decreasing the Lagrange scaler ( $L < 1.0$ ) will emphasize the data error and the inversion will create rougher models that fit the data better.

One should experiment with different values of the Lagrange scaler and decrease it during successive iterations. When multiple iterations are made overnight, for example, automatic Lagrange scaling can be activated by giving a negative value to the *F-option* parameter (e.g.,  $L = -1.0$ ). The automatic value of Lagrange scaler is based on the RMS-error and the given value of *F-option*. The automatic value is always between  $\text{abs}(L)$  and  $\text{abs}(L)/10$ .

Note that the Occam inversion enforces smooth density variations. In real world the density changes are much more abrupt. The Lagrange multiplier sets a global value for the desired model roughness. The fix/free status and the discontinuity parameter provide means to define local variations in the smoothness constraint. The fix/free status imposes certain kind of rigidity around a fixed block. Additionally, discontinuity parameter can be used to define discontinuous interfaces in the model, thus, allowing rapid changes between selected two (or more) blocks.

*G-scale* parameter is used in Occam inversion as global data weight and it is defined separately for the gravity and each gradient components. To change the current data component one needs to use the *Gravity/Gradient* and *Grad. component* buttons. Increasing the value of global data weight increases the importance of the current data set in the inversion. On the contrary, decreasing the *G-scale* value decreases the importance of the data component. The data set can be discarded totally by setting the value of the *G-scale* parameter to zero. Note that currently the data cannot be given individual weights that would depend on the measured data error, for example. This feature may be added in the future.

#### 5.5.1 *Discontinuity information*

The objective of the discontinuity is to define interfaces across which the Occam method will not continue the smoothing. Blocks that are marked with discontinuity information are not added to the mean of the surrounding blocks when the roughness is computed in Occam's method. In practice the discontinuity information is stored as a second parameter in the \*.BLX file with the help of the BLOXER program. The discontinuity is defined using a single (real or integer) value that consists of the sum of following values:

- 0 fully continuous block (normal case)
- 1 fully discontinuous block (special case)
- 2 discontinuous towards positive  $x$  axis (north)
- 4 discontinuous towards negative  $x$  axis (south)
- 8 discontinuous towards positive  $y$  axis (east)
- 16 discontinuous towards negative  $y$  axis (west)
- 32 discontinuous towards positive  $z$  axis (down)
- 64 discontinuous towards negative  $z$  axis (up).

For example, value 126 ( $2+4+8+16+32+64$ ) means that the block is discontinuous in all directions, 30 ( $2+4+8+16$ ) means that the block is horizontally discontinuous and vertically continuous, whereas value 96 ( $32+64$ ) means that the block is horizontally continuous and vertically discontinuous. Likewise, value 2 means that the discontinuity interface is to the eastern side of the block. Usually, in this case the block on the eastern side will have value 4 (discontinuity is on the western side of the block). Discontinuity is stored into the model \*.BLX files as the second parameter after the density. Discontinuity can be edited into the model manually or imported like topography (surfaces, lines, points) using the BLOXER program. The discontinuity information should be based on *a priori* information (or well established assumption). Please, note that this method is still under development.

### 5.6 About computation time

The computation time depends directly on the number of blocks and the number of points where the gravity field is to be computed. Computation of gravity gradients is slightly more time consuming and each gradient component requires some additional time. As an example, consider a model where the number of free blocks is  $N= 34 \times 30 \times 8= 8160$  and the number of data points is  $M= 1020$ . On a PC with 3.2 GHz Intel i5-2500K CPU the forward computation takes about 0,55 seconds using the 64 bit version of GRABLOX2 on a 64 bit Window 7. As a comparison, the computation took 0.76 seconds using the 32 bit version of GRABLOX2.

Although Occam inversion needs to solve much larger matrix system than the SVD, the use of an iterative (CG or QR) solver makes the overall inverse computation much faster. Considering the previous example where the dimension of the linear system is  $9180 \times 8160$ , a single iteration took about 75 seconds to compute (64 bit version). Most of the time is spend redefining the Jacobian, i.e., computing normal equations  $A^T A$ . If there is not enough memory to allocate the auxiliary  $A^T A$  matrix, a QR solver is automatically used. In case of the 32 bit version this typically happens when  $N > 15.000$ . The QR algorithm is computationally more time consuming than CG solver and, hence, it is used merely as an alternative solver. Because of the slowness and the unconstrained nature the SVD inversion it should be avoided when a) the dimension of the inverse problem is greater than  $12000 \times 2000$ , and/or b) some model parameters are fixed based on a priori information. For comparison, a single iteration with the SVD based inversion took about 13 minutes to compute in the example case ( $1020 \times 8060$ ).

## 6. Guidelines for gravity inversion

Numerical inversion of gravity data aims to obtain a model whose synthetic response fits the measured gravity data (and gradient data, if available) with sufficient accuracy. GRABLOX2 is a relatively simple software tool that can be used to determine one or few possible solutions to complicated 3-D gravity problems. The big problem, however, is the non-uniqueness of the inverse solution. One should never consider the inversion results final - they only show possible solutions. Combined geophysical and geological interpretation is always needed to estimate the physical meaning of the inversion model and to decide whether or not the solution is acceptable.

This chapter discusses the various steps needed for the gravity inversion based on a 3-D block model and how they can be accomplished with GRABLOX2 and BLOXER. The reader should note that this chapter applies to forward modelling also. Neither the actual combined geological and geophysical interpretation nor the theory and applications of the gravity method will be discussed.

### 6.1 *Precautions*

Preliminary inspection of the data is used to assess the data quality and sampling and to give some insight to the relation between the gravity anomalies and geological structures. Computer software such as ESRI ArcGIS, Golden Software Surfer or Geosoft/ Oasis Montaj can be used to visualize the gravity, topography, and petrophysical data. Digital geological maps are also useful in the visualization of gravity and petrophysical data.

Before inversion it is also necessary to estimate the dimension of the problem. Although forward computation is quite fast, models with tens of thousands of blocks require lots of computer resources. Nowadays, the amount of physical computer memory rarely hinders forward computation, but it can easily prevent inversion.

### 6.2 *Data preparation*

First of all, one needs to define the gravity data for the inversion. In other words, one needs to create a column formatted data file (\*.DAT) and check its format, dimensions (m, km, ml or ft), and units (mgal, Eo). The data can be evenly discretised or irregularly spaced, and it can include topography or elevation values. Remember, however, that one must choose the study area and the data sampling based on the practical limits of the inversion. For example,

inversion of  $30 \times 30$  data points using a 3-D block model with 10 layers leads to quite a feasible problem of dimension  $M \times N = 900$  data points  $\times$  9000 blocks. Doubling the horizontal discretisation to  $60 \times 60$  points will yield a problem of  $M \times N = 3600$  data points  $\times$  36000 blocks, which requires running the inversion overnight.

Pre-processing (filtering, levelling, corrections) should be made to the data beforehand. Although it is not required, the gridded data should be stored as continuous lines of  $x$  coordinate (from West to East) for each  $y$  coordinate (from South to North). If the data are measured along lines that are not aligned with main geographical coordinate axes, the coordinates should be rotated so that the new data area coincides with the coordinate axes of the model. If the data coordinates cannot be rotated the model must be made larger than data area and a certain amount of the blocks will be wasted in the corners of the model. In that case, one can hide the blocks outside the data area with BLOXER program.

Note that data from a single profile can be interpreted using a 2.5-D model (see chapter 7.2). In that case, the data should be defined so that the profile is aligned with the  $x$  axis or  $y$  axis so that an initial 2.5D model perpendicular to the profile can be constructed (*Compute default* menu item). Note, however, that it is possible to interpret single profile data using a fully 3D block model, although that would be somewhat ambitious.

Secondly, one needs to define the regional field if such data are available. The regional field can be included in the gravity data file or read from a separate file. In the both cases, the regional data should have exactly the same  $x$  and  $y$  coordinates as the gravity data, although it suffices in the latter case that the regional data have the same number of points as the gravity data.

Thirdly, one needs to define the gravity gradient data if such data are available. The gradient data are provided in a separate file, and need not have the same  $(x, y, z)$  coordinates or data extent as the gravity data. The header of the file defines the order in which the gradient (gravity tensor) components are stored in the file. The file can contain just one or several gradient components. The diagonal elements ( $g_{xx}$ ,  $g_{yy}$  and  $g_{zz}$ ) of the gravity tensor are usually the most useful gradient components.

### 6.3 Preliminary operations

After starting up and after opening some previously defined model file (possibly a dummy file if one cancels the file open procedure), the first thing is to read in the gravity data using the

*Read data* item in *File* menu. If regional gravity field is not included in the same data file it can be read in using the *Read regional data* item. Likewise, gradient data is read in using the *Read gradient data* item. The *Map data* and *Profiles* (and *Crossing dir*) buttons can now be used to visualize the data as contour or image maps and profile sections. The *Gravity/Gradient* button changes the current view between gravity and gravity gradient data. The *Grad component* button changes the current view between different gradient components (if available). Note that profile graphs are always interpolated from the (assumed) irregular data.

At this point the user should check the dimensions and use the *Gravity/Scale unit* menu item to choose between meters, kilometres or miles. This is an essential task for correct amplitude of the gravity and gradient computations. In forward modelling one should also check the *Gravity/Computation* menu item. The default option is to compute only the gravity field. Alternatively, one can compute the gravity and the vertical gradient or the gravity and the six components of the gravity tensor ( $g_{xx}$ ,  $g_{xy}$ ,  $g_{xz}$ ,  $g_{yy}$ ,  $g_{yz}$  and  $g_{zz}$ ) and the curvature component ( $g_{uv}$ ). Once the data have been read in the computation mode cannot be changed.

Also, one should check the method used to define the background density using *Background* item in *Gravity* menu. The default option is to use a constant value, for example, the mean petrophysical density (e.g., 2.67 g/cm<sup>3</sup>) or zero, in which case all density values will be relative. Alternatively one can use the mean density of the whole main block or the mean density of each layer, which should work for large crustal models.

Now, if the current model conforms to the problem geometry one can step into chapter 6.6. Otherwise, an initial model needs to be defined.

#### 6.4 Initial model

When creating an initial model the user should first choose between the default (equal height) and the alternative (increasing height) block type using *Edit/Block type* menu item. Since the resolution of the gravity field decreases with increasing depth, the user is advised to use the alternative block type where the block height increases with depth, because the upper parts of the model can be modelled with greater accuracy with the same amount of minor blocks.

If the data are evenly discretised, the *Compute default* item in *Gravity* menu can be used to create an automatic initial model, where the minor blocks are put directly below the (regularly gridded) data points. The horizontal size, position, and discretization will be defined



automatically, whereas the vertical discretization and depth to the bottom will remain the same as before, and, thus, need to be changed manually. If the data contain  $z$  coordinates the surface topography will be adjusted as well. Note that the  $z$  coordinates of the data should be used whether or not topographic correction has been made to the gravity data. If  $z$  coordinates are not provided the data are assumed to locate at level  $z = 0$  and the top of the model is positioned accordingly. The height of the main block should be set manually so that the model does not extent too deep, but it should not be too shallow either. Usually the depth to the bottom should be 0,1–0,5 of the horizontal width of the model. In large-scale crustal studies the depth to the Moho can be imported as a boundary into the model using BLOXER. Note that especially when using gradient data better computational results are usually obtained if the depth to the top of the main block is about half of the width of the blocks. In other words, the model blocks should not coincide with data  $z$  coordinates, otherwise the horizontal size of the individual blocks start to appear in the modelled gradient data. Depending on the data sampling and horizontal size of the blocks the effect of individual blocks can be seen in gravity data as well. Therefore, the discretization should be such that the data points are located above the centres of the blocks.

Note about editing modes: To be able to change the block size and discretization manually, one should use the *Ignore* editing mode. The *Shift* mode, which is the default mode, only allows moving the entire main block (to prevent accidental changes). Note also that the block type (equal/increasing height) cannot be changed when *Shift* mode is active. The *Preserve* mode should be reserved for changes made to an existing model, because it will use 3D interpolation to redefine existing density model inside the new model geometry.

If the data are irregularly spaced, the discretization and fine tuning of the block parameters must be checked and possibly edited manually after applying the *Compute default* button. The BLOXER program can be used to adjust the top topography for a model derived from irregular data. See BLOXER's user manual to learn how to import surface topography.

Practical note: The position and the discretization of the main block should be such that the centres of the blocks locate directly below the (regularly gridded) computation points. This enables maximal relationship with the block density and the gravity field and least amount of effect due to the prism-like shape of the blocks. If the data are regularly sampled this is done automatically for the initial model when *Compute default* option is used.

If the data are highly irregular the abovementioned condition cannot be met. When dealing with regularly spaced data, however, one must remember that the corners of the main block (defined by *X* and *Y-posit* and *X* and *Y-size*) are not located below the corner points of the computation grid but the centres of the minor blocks in the corners.

Because the contour plotting used in GRABLOX2 is not very sophisticated, users are advised to use other programs (e.g., Surfer or Oasis Montaj) to interpolate the data and the *z* coordinates onto regular grids, which are then combined, edited and saved in GRABLOX2 data format (\*.DAT). After this the *Compute default* option can be used to generate the initial model. If the data coverage is not rectangular but it has irregular shape one can first create a regularly discretised initial model using the *Compute default* option. Then one can use BLOXER program to hide the blocks outside the data area in which case they are considered to be equal to background density and omitted from computation and inversion.

The last step in creating the initial model is to define the initial density of the blocks. Normally, the same value is used as for the background density (e.g. 2.67 g/cm<sup>3</sup> or 0 g/cm<sup>3</sup>). The density of the whole super model can be reset by defining the *Param* text field and pressing the *Reset dens* button. When performing large scale modelling the density can be set per layer using the *Edit/Layerwise reset* menu item. Note that resetting does not affect fixed and hidden blocks or their fix/free status.

The initial density (*Param*), background density (*Bg dens*) and the minimum and maximum value of the density range (*Min/Max density*) must also be checked before continuing with the inversion. It is also useful to save the initial model on hard disk using *File/Save model* menu item before the inversion. Note that the computational parameters (scale units, computation and background modes) and the main model parameters (position, size and discretization) are saved into the model file (\*.INP), whereas the information on the individual blocks are saved into a separate block file (\*.BLX). Although the whole initial model can be constructed using the BLOXER program, the computational parameters relevant to GRABLOX2 will not be included into the header of the input file (\*.INP) and they need to be edited manually, preferably based on an existing model file. It is also possible to import density data from other sources or previous investigations into an existing model using the BLOXER program.

Creating the initial model is an important task that greatly affects the inversion results. Once the initial model has been defined the user can step to chapter 6.6 to interpret the data.

If *a priori* petrophysical or topographic data are available and used, then some additional work needs to be done.

#### 6.4.1 *Equivalent layer model*

If the number of layers in the current model is equal to one ( $NZ=1$ ) when the *Compute default* button is applied, GRABLOX2 asks the user whether or not the initial model is going to be an equivalent layer model. Equivalent model, which contains only a single layer of elements, does not aim to resolve the actual density distribution of the subsurface. Because gravity field is a conservative field, the equivalent layer model obtained from density inversion can be used to compute the gravity field on a different elevation level. In other words, equivalent layer model can be used to perform upward continuation of gravity data.

Equivalent layer model can also be used to determine so-called derived Bouguer anomaly from gravity gradient data. In this case, GRABLOX2 asks whether or not the equivalent layer model is going to be based on gradient data instead of gravity data (which is usually irregular and sparsely sampled Bouguer anomaly data). The (sparse) ground gravity data and the (dense airborne) gradient data are inverted together and the resulting equivalent layer model is used to compute the gravity effect at the locations of the gradient data but on the topographic level of the terrain. As a result a densely sampled gravity map that conforms to both the measured gravity field and the gradient data is obtained. Since the gravity map is not based on real gravity data the result is called derived Bouguer anomaly.

For equivalent layer model the program also asks for the height of the layer, the depth to the top of the layer, and the minimum and maximum density value. Usually, the height and the depth to the top of the model should be equal to the minimum size of the (data or model) discretization. The minimum and maximum density values are given unrealistically large values (eg.  $-20$  and  $+20 \text{ g/cm}^3$ ) to prevent the density values getting saturated against the limit values during the inversion. For equivalent layer model the background density is always set equal to  $0 \text{ g/cm}^3$ .

#### 6.5 *A priori data*

Two kind of *a priori* data are normally used. Either the density is known at some point on the surface based on a petrophysical data or the position of a geological contact is known based on geological maps or previous interpretations. It is also possible that the vertical depth to a

geological interface with certain density contrast can also be known based on drilling or other geophysical interpretations.

If petrophysical data are available, it should be imported into the topmost blocks of the model. First, an evenly discretised grid of the density values is made using the PETROCK program or some other suitable program (e.g., Surfer or Geosoft). The gridded data must be stored into a column formatted text file that also contains the x, y, and z coordinates of the data. Secondly, a header \*.HEI file must be created where the column information of the actual petrophysical data file and its name is defined. Then the *File/Import data* menu item of the BLOXER program can be used to import the petrophysical data into the model. Note that the column containing the z coordinates ( $z = 0.0$ ) may need to be added into the file manually (use any spreadsheet program). Be reminded that to enable *Import data* item in BLOXER one must first activate the *Immersion* editing mode. The *alf3/rdis* edit parameter defines the important scale distance, which normally should be equal to the horizontal size of the minor blocks. The *alf1/wght* parameter defines the weight of the existing (old) parameter value. For further information about data importing, please, consult the user manual of the BLOXER program.

Petrophysical data usually correspond only to the top layer. BLOXER's import operation, however, will extend the data to the whole model. Therefore, the density of the blocks below the first layer should be reset to background density and the top layer should be fixed (fix/free = 0) in the initial model. To accomplish this, one can use the interactive editing methods of BLOXER, but it can be done also in GRABLOX2 using *Layer-wise reset* and *Layer-wise fix/free* items in *Edit* menu.

In addition to surface topography, topographic information can also be used to define the vertical z coordinates of internal geological structures such as depth of the groundwater level or the depth to the basement. The depth data can be from wells, drill holes or previous geophysical or geological interpretations. BLOXER can be used to import topography data and fit the height of the blocks automatically. The topography data can be defined on a surface, along 2-D lines, or at individual points. When importing depth values along profiles or at some points, only the nearest blocks will be resized. When importing depth values as a surface, interpolation and extrapolation are used to extend the surface horizontally across the whole model. In these operations, the top or the bottom of the closest block will be shifted to

match the height value. When importing well data, the given layer is forced to fit the defined height (or depth from the surface level).

To finalize the initial model it is sometimes necessary to add margins around it. The purpose of the margins is to reduce edge effects, that is to say the gravity effect due to the density contrast between the blocks and the background density around the model. Because the block model is usually located below computation points, extending its edges outward will move the occurrence of edge-effects outside away from the computation area.

Margins can be added to the model with BLOXER or GRABLOX2 using the *Edit/Add margins* menu item. Usually multiple margins with increasing width (e.g., 100, 200, 500, 1000 m) are added to the model. Note that margins should not be used in normal 3-D density inversions, especially if regional field was defined based on large-scale inversion. Gravity gradient data, however, is more sensitive to edge-effect than the gravity data. Therefore, margins are useful when doing joint inversion of gravity and gravity gradient data.

## 6.6 Gravity inversion modes

Once the measured gravity data has been read in and a suitable initial model has been defined, the user should define the regional field unless it's already included into the gravity data file or it has been subtracted from the gravity data (leaving the residual gravity anomaly). External regional field can be determined in different ways (see chapter 5.1) and is read in using *File/Read regional* menu item. The *Show/Hide regional field* item in *Edit options* sub-menu is used to include it to (or exclude from) map and profile graphs.

The regional field can also be defined using 2D interpolation of manually edited knot points. Knots can be used to define a more complicated regional field than the second degree polynomial of the base anomaly. First, few knot points (minimum 5 points) are added in the map view and then, their level is fine-tuned in profile views. Finally, the *Knots->regional* menu item is used to interpolate a regional field through the knot points. If the regional field needs improving, the level of the existing knots can be adjusted and more knots can be added on other profiles, before the applying the *Knots->regional* item again.

One should also check that the regional field is added into the computed total gravity anomaly (*Gravity field* item in *Gravity* menu). The total field is the sum of the computed response due to the mass distribution of the model blocks, the regional field read from external file, and the base anomaly. Anomalous field is due to the effect of the mass distributions only.

Once the data, initial model and computational parameters have been defined the user can start the interpretation. Usually, the first step is to compute the response of the initial model pressing *Compute* button. Then one can use the *Map data* and *Profiles* buttons to inspect the measured and computed data and their difference. Remember also to check that the dimensions (m vs. km) are defined correctly. Comparing the contour/image map of the data and the layer map of the model one can also see if the model corresponds to the current data. The grid lines of the minor blocks (*Edit options/Grid lines*) and the dots of the data (if labels are shown) shown in contour/image maps of the data can also be used to check proper model discretization.

Often there is a strong difference in the base level of the computed and measured gravity field. This is natural since the measured data are relative and the 3-D modelling does not give absolute gravity values. Thus, the base response should be optimised first.

#### 6.6.1 Base anomaly inversion

Optimization of the coefficients of the base anomaly is a simple, but essential part of the interpretation process, particularly if no other means are used to define the base level or regional trend. The base anomaly is defined as a polynomial of the second degree. Base anomaly optimization is selected from the uppermost list widget on the right control panel. The second list widget is then used to define the combination of the coefficients of the base anomaly that will be optimised:

- |                      |   |
|----------------------|---|
| 1. <i>None</i>       | the base response is not optimised (default),               |
| 2. <i>Base A</i>     | only the base level ( <i>A</i> ) is optimised,              |
| 3. <i>X grad B</i>   | only the x gradient ( <i>B</i> ) is optimised,              |
| 4. <i>Y grad C</i>   | only the y gradient ( <i>C</i> ) is optimised,              |
| 5. <i>X2 grad D</i>  | only the <i>D</i> coefficient is optimised,                 |
| 6. <i>Y2 grad E</i>  | only the <i>E</i> coefficient is optimised,                 |
| 7. <i>XY grad F</i>  | only the <i>F</i> coefficient is optimised,                 |
| 8. <i>Linear ABC</i> | all linear coefficients are optimised ( <i>A, B, C</i> ),   |
| 9. <i>All</i>        | all coefficients are optimised ( <i>A, B, C, D, E, F</i> ). |

If the difference between the measured and computed data is considerable, one should first optimize only the base level. After selecting the base response optimization and *Base A* as a combination from the pull-down list widgets below *Optimize* button, one needs to press the *Optimize* button to do the inverse iterations, the number of which are defined by the *Iters* text field. Before inversion one forward computation of the whole model is also performed.

Usually, the change in the fit between measured and computed data due to the base response optimization is best seen in profile response graphs. Once the base level is fitted close enough one can include the  $x$  and  $y$  gradients into the optimization (*Linear ABC*). Note that in 2.5-D inversion if the data are defined along  $x$  and  $y$  directed profiles only the  $x$  or  $y$  dependent coefficients ( $A$ ,  $B$ , and  $D$  or  $A$ ,  $C$  and  $E$ ) can be optimised.

The optimization of the base anomaly tries to fit both positive and negative gravity anomalies to minimize the data misfit. Negative gravity anomalies, however, require negative density contrast corresponding to mass deficiency ("negative mass"). Therefore, after optimization of the base anomaly, especially when using absolute (*a priori*) density values, it is necessary to adjust (usually decrease) the base level of the base anomaly so that mainly positive gravity anomalies remain.

Note about base level inversion: The base anomaly can be biased towards the bottom level of the gravity anomaly by increasing the *I-option* parameter in the left control panel. The bigger its value (IOPT= 1–10) the further downwards the base level will be positioned in base level inversion. Negative value of *I-option* parameter will shift the base anomaly towards the top level of the gravity anomaly, which is useful when interpreting sediment formations with negative density contrast. Note that the biasing of the base anomaly does not work when base level is optimised together with density values in Occam inversion.

Finally, note that the menu item *Comp -> Regi* can be used to cast the current computed data onto the regional data. If the model is a so-called zero-model, that is, the block densities are equal to the background density or the main block is temporarily located very deep, the computed data will be equal to the base response ( $g_c = g_b$ ). By using the *Remove regional* menu item one can subtract the base anomaly from the measured data. In most cases this will clarify the inspection of the contour and image maps of the gravity data and will bring up more details. See chapter 5.1 for more information about regional field.

After the base level is defined the actual 3-D density inversion can start.

#### 6.6.2 Optimize base to knots

Sometimes fitting the base anomaly to the gravity data (with or without biasing towards the root or the top of the gravity anomaly) does not produce satisfactory results. Knots are used to define the regional field by interpolation, but they can also be used as user defined gravity

points to which the base anomaly should be fitted. This is accomplished activating the *Optimize base to knots* check box at the bottom of the left control panel.

Instead of fitting the base anomaly to the gravity data it will be optimised to the knots that should be evenly distributed across the data area. Knot points (minimum 5 points) are first added in the map view and their level is then fine-tuned in profile view. After base anomaly inversion the fit between the measured gravity and the base anomaly is checked across different profile views. If the base anomaly needs improving, the level of the existing knots is adjusted and/or more knots are added, and the base anomaly inversion is performed again.

### 6.6.3 *Density inversion*

Once the gravity data and the initial model have been defined and the base anomaly and the possible regional data have been set by optimization or by other means, one can proceed with the density inversion. The objective of the 3-D gravity inversion is to find out the absolute or relative density variations below the study area where the gravity field has been measured. Considering a 3-D block model this means the optimization of the density of the minor blocks so that the data error, i.e., the difference between the measured and the computed data, is minimised.

Two different density inversion methods are currently used in GRABLOX2. The unconstrained *Density* inversion uses the old SVD based algorithm and should be used only when using rather small models. Because *Density* inversion is an unconstrained it should be used only if there are no fixed blocks even if additional constraining is used. The *Occam d* inversion is a constrained inversion method that minimizes the model error (roughness of the model) together with the data error. The *Occam d* inversion is preferred because it can better utilize *a priori* (petrophysical) density information.

To perform full density inversion one needs to run several iterations. If the inversion takes lots of time, multiple iterations should be run overnight or over weekend. In *Occam d* inversion the initial value of the Lagrange multiplier should be set equal to the default value ( $L = 1.0$ ) so that the densities get better adjusted with fixed blocks. The L-value can be decreased at later iterations to produce better data fit. Very small L-value ( $L < 0.1$ ) should be avoided, because it can disturb the smoothness of the density of upper parts of the model. Automatic L-value can be set by giving it a negative value.



When using petrophysical *a priori* data to fix the density of the surface layer (or some parts of it) one can optimize the base level of the base anomaly together with density. Because depth weighting always puts certain amount of mass to the bottom of the model the density values of the free blocks may not get adjusted properly with the fixed density values of the surface if base level is fixed. In principle, it would be advantageous if the initial density of the free blocks is equal to the background density which is equal to the minimum density, because the inversion would then need to generate only positive gravity effects and positive density contrasts. However, the use of "negative" mass that appears when minimum density is smaller than the background density allows faster convergence. By changing the regional level the user can try to affect to the abundance of positive and negative masses.

The models resulting from block model inversion, particularly from the Occam inversion, are rather smooth and special care must be taken when assessing the actual properties and density values of the geological models that they are interpreted to represent. Also, the user must always remember that gravity inversion has no unique solution – an infinite number of possible models can fit the measured data equally well. To test the validity of the model, the density values and fix/free status can be edited manually using the BLOXER program and the differences in the fit of the various models can be used to assess the limiting values of density, depth and depth extent that are usually the most interesting quantities of the interpretation.

Due to the nature of gravity data one should bear in mind that the horizontal variation of the density distribution is easier to interpret, but the depth to the top and the depth extent of the targets are very difficult to interpret using a block model. For this purpose, the results of 3-D block model interpretation can be used as an initial model for the interpretation programs that use parametric models (e.g., Modelvision). Migration of 3-D block models to parametric models might be implemented in future GRABLOX2 or BLOXER versions.

### 6.7 *Post-processing*

As discussed in the beginning of this chapter, the user must finally decide whether the resulting inversion model is physically and geologically acceptable or not. Due to the non-uniqueness of the inverse problem, the resulting model is only one of the many possible solutions. At the moment GRABLOX2 does not provide any detailed information about the resolution of the inverse solution. The user can try to assess the validity of the solution by making changes into the model and investigating how the changes affect the computed gravity. Moreover, by using a completely different initial model one can try to estimate the

robustness of the solution. Often this part of the interpretation process is the most time consuming.

After the actual interpretation has been made the user would probably like to present the results graphically. The plotting capabilities and the quality of GRABLOX2 and BLOXER printing are quite poor. Users are advised to use external plotting programs such as Surfer or Geosoft Oasis montaj to prepare maps for published reports. The synthetic data computed for the interpretation model and the measured data are saved in column formatted text files using the *File-Save data* menu item. The inversion results (information text) and the synthetic data will be saved in text files using *File-Save results* menu item.

Note about forward computation: When a model based on a previous inversion is opened the forward computations are made on a regular grid using a constant  $z$  coordinate value (height), which is equal to the top of the main block or the value given by *Z-level* parameter. So, if the original data (on which the model was based) were irregularly spaced or the data had  $z$  coordinates the computed gravity anomaly will be different from the previous inversion results. Therefore, to re-compute the synthetic data at correct data position and elevation, the measured gravity data (and regional field and gradient data) should also be read in.

Although the block data are stored in column formatted text files (\*.BLX), they have a format of their own, where the  $z$  coordinates are attached to the top of the minor blocks. Moreover, if the alternative block type is used, the  $z$  coordinates are not equally spaced. Thus, to plot the model data with external software one should use BLOXER's *File/Export data* menu item to interpolate the block data on an evenly sampled grid. The export operation shifts the  $z$  coordinates from the top of the block to the middle of the block and adds additional layers when the alternative block type used. A special output format allows saving the data in a format suitable for Fortner's Transform T3D program that can be used to create more advanced 3-D views of the density data with transparency and perspective views. Voxler by Golden Software is another recommended 3D visualization software.

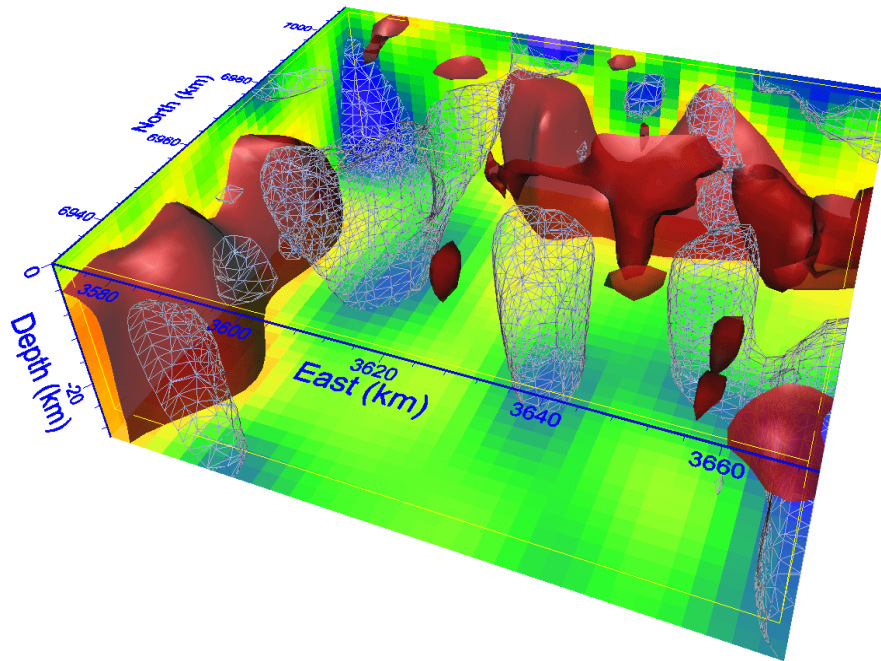


Figure 6.1 Example of Voxler's visualization of a 3D density model: solid red iso-surfaces delineate high density areas whereas low density areas are shown by mesh lines only.

## 7. File formats

### 7.1 Model files

The following example illustrates the format of the input model \*.INP file:

```

4      2.12
-250.00    -250.0    250.00    250.00    50.0    25.00    .000
-250.00    -250.0    250.00    250.00    50.0    25.00    .000
0      0      0      0      2      0      0      0
1.00    10.00    .000    .000    .000    .000    .000    .000
-275.000    -262.500    0.000    550.000    525.000    220.000
11     21      8
1      1      0
1
1848      2      1
0  0.100000E+01  0.000000E+00  0.300000E+01  0.100000E+01
0      0      0  126      1
0

```

- Line 1 defines the number of lines (NOL=4) that are specific to the GRABLOX2 and the version number of the model file (VER=2.12). Depending on the version number GRABLOX2 reads the following NOL lines a bit differently. Note, that GRABLOX2 dependent lines will be skipped over by BLOXER.
- Line 2 defines the computation grid for gravity data:  $X$ -start &  $Y$ -start (-250, -250),  $X$ -end &  $Y$ -end (+250, +250),  $X$ -step &  $Y$ -step for gravity (50, 25) and the  $Z$ -level of computation (0). Note that the data elevation is defined as a positive value if the level is above the surface ( $z = 0$ ).
- Line 3 defines the computation grid for gravity gradient data. In this example the gradient data grid is the same as that of gravity data.
- Line 4 defines some computational options: 1) dimension (M2KM, 1= meters, 2= kilometres, 3= miles, 4= feet), 2) computation mode (IGRA, 0= block gravity, 1= point gravity, 2= vertical gradient), 3) gravity field (ITOT, 0= anomalous field, 1= total field), 4) background density (IREG, 0= given value, 1= mean layer density, 2= mean-block density), and 5) parameter roughness (IPAR, 1= not defined, 2= defined as a second parameter). The remaining three parameters are reserved for future use.
- Line 5 defines the background density ( $d_0$ ,  $\text{g/cm}^3$ ), and the coefficients ( $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ ,  $F$ ) and the shifted origin ( $x_0$ ,  $y_0$ ) of the second degree polynomial defining the gravity base trend. Note that  $B$  and  $C$  have been multiplied by 1000 ( $\text{mgal/km}$ ) and  $D$ ,  $E$  and  $F$  are multiplied by  $10^6$  ( $\text{mgal/km}^2$ ).

- Line NOL+1 defines the  $x$ ,  $y$  and  $z$  coordinates of the top-south-western corner of the main block ( $X0$ ,  $Y0$ ,  $Z0 = -275.0$ ,  $-262.5$ ,  $0.0$ ), and the size of the main block in  $x$ ,  $y$  and  $z$  directions ( $dX$ ,  $dY$ ,  $dZ = 550$ ,  $525$ ,  $220$ ). Note that the depth ( $Z0$ ) is defined as a positive value if it is below the surface ( $z = 0$ ).
- Line NOL+2 defines the discretisation, i.e., the number of blocks in  $x$ ,  $y$  and  $z$  directions ( $Nx$ ,  $Ny$ ,  $Nz = 11$ ,  $21$ ,  $8$ ).
- Line NOL+3 defines some properties of the block file: The first parameter defines the block (\*.BLX) file format (IFO); 0 = \*.BLX file does not exist (the block model will be generated automatically), 1 = normal text file and real (floating point) values, 2 = normal text file and 2-byte integer values, 3 = binary file and real (floating point) values, 4 = Binary file and 2-byte integer values, 5 = binary file and 2-byte integer values, the byte order of which are reversed when the data file is being read (allows Unix ↔ PC conversion). The second parameter defines the block type; if IBO = 1 all blocks are assumed to have equal height (default), if IBO = 2 the block height is increasing with depth, and if IBO < 0 then block reduction has been used (see BLOXER documentation). The 3rd parameter is reserved for future use.
- Line NOL+4 defines the zooming level (IZO = 1 (un-zoomed), 2, 3, ..., 10). If the zooming level were bigger than 1, the next line would contain the  $x$ ,  $y$ , and  $z$  coordinates of the centre of the zoom-block, and the dimension of the zoom-block in  $x$ ,  $y$  and  $z$  directions. GRABLOX2 does not utilize the zoom options at all.
- Line NOL+5 (in this case) defines the number of blocks (NOB = 1848), the number of parameters (NOP = 2), and the index number of the column (ICO = 1) that contains the density values to be read in from the \*.BLX file. Note that if block reduction had been used and the block file would have been saved using the *File-Save red. model* menu item, the number of actual minor blocks might be smaller than the value based on discretization.
- Line NOL+6 contains the parameters that define some properties of the first parameter of the block model: 1) scale type: 0 = linear scale (for density), 1 = logarithmic scale (used for resistivity, for example), 2) default value ( $1.0 \text{ g/cm}^3$ ), 3 & 4) minimum and maximum value that are used for the colour scale and inversion limits ( $0$  &  $3.0 \text{ g/cm}^3$ ), and 5) scaling factor (multiplier) used for labels in layer and section views (1).
- Because parameter roughness was included in the model (IPAR = 2), line NOL+7 defines the properties of the roughness used in BLOXER. For roughness these parameters are

fixed (0= linear parameter, 0= default value, 0= minimum, 126= maximum, and 1= scale value). This line would be omitted if (IPAR= 1).

- The last line defines the number of knots (IKNOT= 0) used for a manually defined regional field. If the file would contain knots, then the remaining lines would contain the X (east), Y (north) coordinates and gravity value at the knot location (mgal), and two index number that define the location of the knot on X and Y sections. The total number of knots is limited to 100.

When gravity gradients are computed the model file is slightly different. If the model is made for vertical gradient computations there will be an additional line after the one that defines the base anomaly line of the gravity data. Thus, the number of GRABLOX2 dependent lines is NOL= 5. The additional line defines the global weight (*G-scale*) and the base anomaly parameters of the vertical gradient data. If the model is made for full gravity tensor computations there will seven additional lines. The rows would correspond to the  $g_{xx}$ ,  $g_{xy}$ ,  $g_{xz}$ ,  $g_{yy}$ ,  $g_{yz}$ , and  $g_{zz}$  tensor components and the  $g_{uv}$  curvature component. As with vertical gradient case, the first parameter is the global weight (*G-scale*) of each gradient data component and the remaining columns define the coefficients of the second degree polynomial for the regional base level trend for the gradient.

If vertical gradient is to be modelled, the top of the model file would be like this:

5	2.12								
-250.00	-250.0	250.00	250.00	50.0	25.00	.000			
-250.00	-250.0	250.00	250.00	50.0	25.00	.000			
0	1	0	0	2	0	0	0		
1.00	10.00	.000	.000	.000	.000	.000	.000	.000	.000
1.00	.000	.000	.000	.000	.000	.000	.000	.000	.000
etc	...								

In principle there is no need to edit model files manually. However, if the \*.BLX file is missing one can still open the model by resetting the file format parameter (IFO= 0) on line NOL+3, so that a dummy block model will be generated automatically. Also, if the \*.BLX file was generated using reverse binary INT\*2 format of Unix/Sun/Mac, the model can be opened by resetting the file format parameter (IFO=5). Note that binary files with real (floating point) values cannot be transported across different computer platforms.

The \*.BLX file has simple column format, where the columns correspond to: the x, y, z size of the blocks (columns 1–3), the x, y, z position of the blocks (columns 4–6), the fix/free/hidden/reduced block status (column) 7, and the parameter columns (8–17). Normally, the density

values are in the 8.th column. If discontinuity information is utilised it must be defined as the second parameter so it is stored next to density values (9.th column). The horizontal ( $x$ ,  $y$ ) position of a minor block is fixed to the centre of the rectangular block area, but its vertical  $z$  position is fixed to the top of the block. Moreover, for the block model the positive  $z$  axis points downwards so normally the  $z$  coordinates of the blocks are positive. This may be somewhat misleading since the data  $z$  coordinates (topography and elevation) increase upwards. Note that the order of the do-loops used to save the model data is:  $z$  loop,  $y$  loop,  $x$  loop. This means that for each layer (from top to bottom) the blocks are saved so that for each  $y$  coordinate value (from South to North) the data is saved along  $x$ -axis (from West to East). Please, see the user's manual of the BLOXER program for additional information considering the block model.

Note that when *Autosave* option is enabled, the model files (\*.INP & \*.BLX) are saved during each iteration so that the user can study them afterwards. The files are named GRABLOX2\_AUTOFILE\_XXX.\*, where XXX tells the number of iteration. These files are saved in the current working directory. Even if *Autosave* is not enabled, the model of the last iteration is saved in the program folder. These model files are named as GRABLOX2\_CURRENT\_ITER.\*.

## 7.2 Gravity data files

The default extension of gravity data files is \*.DAT. Interactive input of gravity data from generic column formatted text files is possible if the first character on the first line is a one of the comment characters; "!", "#" or "/". In this case the user is asked to provide all the necessary column information interactively. Otherwise, the file is assumed to contain a header that defines how to read the file directly without user intervention.

The example below illustrates the format of a \*.DAT file used for (measured) gravity data:

```
Example gravity data
153 1 2 0 3 0

-200.00 -200.00 0.2007733E-01
-150.00 -200.00 0.2855821E-01
-100.00 -200.00 0.3860382E-01
-50.00 -200.00 0.4719916E-01
  0.00 -200.00 0.5056060E-01
etc ...
```

The first line is a header that will be used in the response graphs as a secondary title. The header line can be empty. Lines starting with "!", "/", or "#" characters will be considered as comment lines. The 3.rd line contains six integer values:

1. The total number of data points,  $M$
2. The column index of the  $x$  (east) coordinate,  $ix$
3. The column index of the  $y$  (north) coordinate,  $iy$
4. The column index of the  $z$  (topography) coordinate,  $iz$
5. The column index of the gravity data,  $id$
6. The column index of the regional data,  $ir$ .

The order of data columns can be arbitrary. The columns of  $x$  and  $y$  coordinates, however, must be defined before actual data columns. If the column index of the  $z$  coordinate is zero then the data are assumed constant value defined by the *Z-level* parameter in the left control panel.

Profile data can be defined using three alternative methods. If the column index of the  $x$  or the  $y$  coordinate is zero ( $ix = 0$  or  $iy = 0$ ), the corresponding coordinate is considered to be missing and the data will be put along the  $x$  axis ( $iy = 0$ ) or the  $y$  axis ( $ix = 0$ ). For interpretation purposes one should reformat data files to pass the profile distance coordinate as  $x$  or  $y$  coordinates so that an default initial model can be computed automatically. For modelling purposes a generic measurement profile from  $(x_1, y_1)$  to  $(x_M, y_M)$  is assumed if the column indices of the  $x$  and  $y$  coordinates have opposite signs. Thus, if the example would read "153 1 -2 0 3 0", the data would be handled as general single profile data ( $Y\text{-step} = 0$ ).

The abovementioned file format is used also when data are saved into a file using the *Save data* item in *File* menu. Note that the resulting order of the columns depends on the existence of measured data and regional field, which are also added to the saved data file.

### 7.3 Gradient data files

The default extension of gravity gradient data files is \*.GAT. Interactive input of gravity gradient data from generic column formatted text files is possible if the first character on the first line is a one of the comment characters; "!", "#", or "/". In this case the user is asked to provide all the necessary column information interactively. Otherwise, the file is assumed to contain a header that defines how to read the file directly.



Example gradient data

# X, Y, Z, Grav, Gxx, Gyy, Gzz

231 1 2 3 7 3  
5 0 0 6 0 7

-250 -250 20 0.100494E+02 0.733776E+00 0.273901E+01 -0.347278E+01  
-200 -250 20 0.100644E+02 -0.137107E+01 0.552706E+01 -0.415600E+01  
-150 -250 20 0.100799E+02 -0.431615E+01 0.908393E+01 -0.476777E+01  
-100 -250 20 0.100921E+02 -0.692114E+01 0.121797E+02 -0.525863E+01  
-50 -250 20 0.100974E+02 -0.795603E+01 0.135430E+02 -0.558699E+01  
0 -250 20 0.100951E+02 -0.734493E+01 0.130158E+02 -0.567088E+01  
etc ...

The first line is a header that will be used in the gradient response graphs as a secondary title. The header line can be empty. Lines starting with "!", "/", or "#" characters will be considered comment lines. The fifth line (in this case) contains six integer values that are:

1. The total number of data points,  $M$
2. The column index of the  $x$  (east) coordinate,  $ix$
3. The column index of the  $y$  (north) coordinate,  $iy$
4. The column index of the  $z$  (topography or elevation) coordinate,  $iz$
5. The total number of data columns (including coordinates),  $it$
6. The number of data columns to be read in,  $id$

The next line defines the column indices of the gradient data corresponding to the six tensor components ( $1 = g_{xx}$ ,  $2 = g_{xy}$ ,  $3 = g_{xz}$ ,  $4 = g_{yy}$ ,  $5 = g_{yz}$ ,  $6 = g_{zz}$ ) and the curvature component ( $7 = g_{uv}$ ). Column index equal to zero means that the corresponding component is omitted. Thus, the example above defines  $g_{xx}$ ,  $g_{yy}$  and  $g_{zz}$  components in columns 5, 6 and 7. Note that the order in which the components need to be defined is fixed, but the columns themselves can be in arbitrary order.

A special case is reserved for the vertical gradient data. In this case the second parameter line ("5 0 0 6 0 7") is not needed and the fifth line could read " 231 1 2 3 7 1". Thus, if the total number of gradient data columns is set equal to one ( $id=1$ ), the file is assumed to contain only vertical gradient which will be read from the column indicated by the total number of columns ( $it$ ). Note, however, that the format shown in the example can be used to define vertical gradient data as well (data columns would be "0 0 0 0 0 7").

Profile data can be defined using two alternative methods. Firstly, if the column index of the x or the y coordinate is set zero ( $ix = 0$  or  $iy = 0$ ), the corresponding coordinate is considered to be missing and the data will be put along the x axis ( $iy = 0$ ) or the y axis ( $ix = 0$ ). Secondly, data are assumed to correspond to a straight measurement profile from  $(x1, y1)$  to  $(xM, yM)$  if the column indices of the x and y data have opposite signs. Thus, if the example would read "231 1 -2 3 7 3", the data would be handled as general single profile data ( $Y\text{-step} = 0$ ).

GRABLOX2 cannot be used to interpret gradient data without any gravity data. Although gravity and gradient data can be defined independently (different x and y coordinates, sampling and elevation) they both need to be defined consistently. This means that both gravity and gradient data need to be defined either along two profiles or two 2D map areas. Note also that the data extent ( $X\text{-start}$ ,  $Y\text{-start}$ ,  $X\text{-end}$ ,  $Y\text{-end}$ ) and the computation of the default initial model (or equivalent model) can be based on gradient data too.

#### 7.4 *Output files*

Information about the model and system parameters used in the computation are stored in the results file (\*.OUT). The result file also contains information about the inversion. Since the file format is self-explaining it will not be described here.

## 7.5 Graph options

Several graph related parameters are defined in the GRABLOX2.DIS file. One can edit the \*.DIS file manually, restart the program or use the *File/Read disp* menu item make the changes visible. If the format of the file should get corrupted, one should delete the \*.DIS file and a new one with default values will be generated automatically the next time the program is started. Except for the top part of the file shown below, the format of GRABLOX2.DIS file is self-explaining since it defines the various text strings used in the graphs.

```
Grablox 2.0 graph parameters
```

```
  30   27   24   20   16
   1    1    1    0    0    1    1    1
300 460    0    1    1    0
0.60 0.85 0.00 0.00 0.00 0.00
200.  30.   4.
```

```
Gravity modelling
Gravity anomaly
Gravity gradient
Computed
Measured
Regional
Difference
X / East
Y / North
Z / Depth
Distance
Density (g/cm3)
Log10 (Density)
(mGal)
(Eötvös)
```

- The first line is a title that defines the version number.
- The second line defines the five character heights used for: 1) the main title and the graph axis titles, 2) the axis labels, 3) the plot legend text, 4) the model description text, and 5) the axis labels in the 3-D model view.
- The next line contains some integer values that modify the graph appearance: 1) include/exclude (1/0) the information text in the graphs, 2) include/exclude (1/0) the small 3D model view in the graphs, 3) define the corner where the legend text will be positioned, values 1-4 put the legend in SW, SE, NE or NW corner of the page (outside the graph), values 5-8 put the legend in the SW, SE, NE, or NW corner inside the graph, 4) set the colour scale: 0= rainbow, 1= reverse rainbow, 2= grayscale, 3= reverse grayscale, and 4= temperature (red) scale, 5) set normal/widescreen (0/1) display mode, 6) show/hide (1/0)

labels in layer maps, 7) show/hide (1/0) grid lines in data maps and layer maps, 8) show/hide (1/0) the section in profile views (and profile in section views).

- The next line contains more integer values that modify the graph appearance: 1)  $x$  (horizontal) offset (in pixels) of the origin from the left side of the page, 2)  $y$  (vertical) offset (in pixels) of the origin the bottom of the page, 3) create either contour or image maps (0/1), 4) show/hide (1/0) regional field when data maps are shown, 5) show/hide (1/0) data error (difference) when data maps are shown, 6) display the difference as absolute values or as percents (0/1).
- The next line with numerical values defines: 1) the length of the longest axis relative to the size of the remaining (origin shifted) width and height of the plot area. The 4.th line defines horizontal and vertical viewing angles and a (relative) viewing distance used in the 3-D model views.
- The last line with numerical floating point values defines the horizontal and vertical angle and the relative distance of the view point for the 3D model view.
- The remaining lines define various character strings for the graphs. The maximum length of the strings is 70 characters. Note the special use of caret "^" and tilde "~" symbols for displaying the superscript and subscript (exponent) in unit ( $\text{g/cm}^3$ ) and 10-base logarithm ( $\log_{10}$ ). The dollar sign "\$" sets the text level back to normal. See DISLIN manual for further information on special text formatting such as Greek symbols.

## 8. Additional information

I wrote the first GRABLOX version in 2003 when I worked at the Geological Survey of Finland in the 3-D crustal model (3DCM) project funded by the Academy of Finland. Because of the many requests I made GRABLOX version 1.5 freely available in 2005. Several fixes and updates to the computation, inversion and graphical user interface were implemented in version 1.6 released in 2008. GRABLOX2, initially released in December 2009, was largely rewritten to comply with gravity gradient data. To keep things simple I decided to leave the block height inversion out from GRABLOX2.

The forward computation is based on the GPRISM algorithm by Sven-Erik Hjelt (1974). The equations for gravity gradients are provided Zhang et al. (2000) but the algorithm is adapted from Hjelt's (1972) solution for the magnetic field of a magnetised prism. The idea of depth weighting has been discussed by Li and Oldenburg (1996 & 1998). The linearised inversion has been described by Jupp & Vozoff (1975) and Hohmann & Raiche (1988). The adaptive damping has been described in my PhD thesis (Pirttijärvi, 2003). The constrained Occam inversion is based on the paper by Constable et al. (1987). The SVD and the conjugate gradient solver algorithms have been adapted from Press et al. (1988). The QR decomposition algorithm, originally by Lawson & Hanson (1974), was found from Alan Miller's webpage (<http://users.bigpond.net.au/amiller/>).

The program is written mostly in standard Fortran 90 but it contains some extensions of Intel Fortran (version 15). The user interface and the graphics are based on DISLIN graphics library (version 10.2) by Helmut Michels (<http://www.dislin.de>). Because DISLIN is independent from the operating system the program can be compiled and run on other operating systems (Linux or Mac OS X) without major modifications. However, the source code of GRABLOX2 is not publicly available at the moment. If you have suggestions for improvements, please, inform me.

Because GRABLOX2 is under development this user guide may not describe all the latest changes and additions made to the software. Additional information may be obtained from my website of free geophysical software at: <https://wiki oulu.fi/x/EoU7AQ>.

### *8.1 Terms of use and disclaimer*

You can use GRABLOX2 free of charge. If you find the program useful, please, let me know. It would be nice to hear that someone has found my software useful.

If you publish results obtained using GRABLOX, please, use this user's manual as a reference. For example, "Pirttijärvi, M., 2014. *GRABLOX2 - Gravity interpretation and modelling using 3-D block models, User's guide to version 2.1*. University of Oulu, 62 pp." Optionally, you can make link to website at: <https://wiki.oulu.fi/x/EoU7AQ>.

GRABLOX2 software is provided as is. The Author, University of Oulu and Geological Survey of Finland disclaim all warranties, either expressed or implied, with regard to this software. In no event shall the Author, University of Oulu and Geological Survey of Finland be liable for any indirect or consequential damages or any damages whatsoever resulting from loss of use, data or profits, arising out of or in connection with the use or performance of this software.

## 9. References

- Blakely, R.J., 1995. Potential Theory in Gravity and Magnetic Applications. Cambridge University Press. 441 pp.
- Constable, S.C., Parker, R.L. & Constable, C.G., 1987. Occam's inversion - A practical algorithm for generating smooth models from electromagnetic sounding data, *Geophysics*, 52, 289–300.
- Hohmann G.W. and Raiche A.P., 1988. Inversion of controlled-source electromagnetic data. In: Nabighian, M.N. (Ed.) *Electromagnetic methods in applied geophysics*, Volume 1, Theory. Soc. of Expl. Geophys., Tulsa, p. 469-503.
- Hjelt S.-E., 1972: Magnetostatic anomalies of dipping prisms. *Geoexploration*, 10, 239-254.
- Hjelt S.-E., 1974. The gravity anomaly of a dipping prism. *Geoexploration*, 12, 29-39.
- Jupp D.L.B and Vozoff K., 1975. Stable iterative methods for the inversion of geophysical data. *Geophys. J. R. Astr. Soc.* 42: 957-976.
- Lawson, C.L. & Hanson, R.J., 1974. Solving least squares problems. Prentice-Hall.
- Li, Y. and Oldenburg, D.W., 1996. 3-D inversion of magnetic data. *Geophysics*, 61, 394-408.
- Li Y. and Oldenburg D.W., 1998. 3-D inversion of gravity data. *Geophysics*, 63, 109-119.
- Pirttijärvi M., 2003. Numerical modeling and inversion of geophysical electromagnetic measurements using a thin plate model. PhD thesis, Acta Univ. Oul. A403, Univ. of Oulu.
- Pirttijärvi M., 2005. BLOXER - Interactive visualization and editing software for 3-D block models, User's guide to version 1.6c, University of Oulu, 54 pp.  
<https://wiki oulu.fi/x/eYU7AQ>
- Press, W.H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T., 1988. Numerical recipes, the art of scientific computing. Cambridge University Press.
- Zhang C., Mushayandebvu M.F., Reid A.B., Fairhead J.D. and Odegard M.E., 2000. Euler deconvolution of gravity tensor gradient data. *Geophysics*, 65, 512–520.